LIMIT CYCLES FOR A CLASS OF CONTINUOUS-DISCONTINUOUS PIECEWISE DIFFERENTIAL SYSTEMS

BILAL GHERMOUL[1] AND JAUME LLIBRE[2]

Abstract. During this century an increasing interest appeared for studying the planar piecewise differential systems, this is due to their big number of applications for modelling many natural phenomena. For understanding the dynamics of the planar differential systems we must control the existence or non-existence of periodic orbits and limit cycles. So many papers have been published studying the existence or non-existence of periodic orbits and limit cycles for continuous or discontinuous piecewise differential systems. But until now very few papers study the periodic orbits and limit cycles of piecewise differential systems which in a piece of the line of separation between two differential systems of the piecewise differential system this be continuous and in another piece be discontinuous.

We study the periodic orbits and limit cycles of the planar continuous-discontinuous piecewise differential systems separated by two parallel straight lines, such that either in one of these straight lines the piecewise differential system is continuous and in the other one discontinuous. In two pieces of these piecewise differential systems there are arbitrary Hamiltonian systems of degree two and in the third piece there is an arbitrary Hamiltonian systems of degree one forming the continuous-discontinuous piecewise differential systems. We determine the limit cycles of these piecewise differential systems by considering two cases, in the first the Hamiltonian system of degree one can be in the middle of the three zones, and in the second can it is in one side of the three zones.

1. Introduction

Poincaré's works started the qualitative study of the differential systems instead of finding exact or approximative solutions of themselves. With him also appeared the notion of the limit cycles which became one of the most important objects for understanding the dynamics of the differential systems in the plane, see [22].

The concept of a limit cycle is a concept whose importance is not hidden from any researcher in the area of the differential systems in dimension two and related fields. But in general to determine the existence or absence of a limit cycle is not an easy task. See for instance the Hilbert's 16th problem [9, 10, 11].

At the beginning of the 1930s the limit cycles started to be studied in the continuous and discontinuous piecewise differential systems due to their importance in many mechanical and electrical applications, for more information on their past and present applications, see the books [1, 5, 23] and the survey [21]. The continuous piecewise differential systems have been studied for several authors, see for instance [19, 20, 4, 8, 14], and for the discontinuous ones see without being exhaustive [2, 3, 8, 12, 13, 15, 16, 17, 18].

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