PHASE PORTRAITS OF COMPLETELY SYMMETRIC CENTERS

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ABSTRACT. It is well known that the existence of symmetries are important in many mathematical and physical phenomena. Thus the existence a symmetry for a given analytic differential system in the plane can determine weather a monodromic point is a center. In this paper we classify the phase portraits of the so-called completely symmetric vector fields. More precisely, we prove that the phase portraits of the completely symmetric vector fields are two kind of global centers in the Poincaré disc.

1. INTRODUCTION

The use of symmetries in the differential systems has a long history, thus Birkhoff in 1995 already used them in his work on the restricted three-body problem [1]. The symmetries have many applications in mathematics, physics, in many natural phenomena, in areas such as thermodynamics and quantum mechanism, see for instance [3]. And symmetries arouse a lot of interest even today.

Special types of symmetries have interesting applications in a classic problem known as the *center-focus problem*, i.e. how to distinguish when a monodromic singular point is a focus or a center. In planar analytic differential systems the presence of a reversible symmetry guarantees that a monodromic point is a center (see definitions in Section 2.1).

In [7] Zhitomirski gaves a classification, via orbital equivalence, of all completely symmetric centers defined by vector fields according to the type of its 2-jet.

In this paper we shall classify the phase portraits in the Poincaré disc of the completely symmetric centers described in [7].

We show that there exist only two distinct phase portraits for the completely symmetric centers, they are given in Figure 1(a) and Figure 1(b).

The main result in this paper is the following:

Theorem 1. *The phase portrait in the Poincaré disc of any completely symmetric center of a polynomial differential system is topologically equivalent to one of the two centers of Figure 1.*

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