LIMIT CYCLES FROM THE PERTURBATION OF A QUADRATIC UNIFORM ISOCHRONOUS CENTER INSIDE THE DISCONTINUOUS PIECEWISE QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. Piecewise differential systems in the plane have been extensively studied in the last two decades, due to the vast application of these systems to describe natural phenomena. The knowledge of the existence or not of periodic solutions, in particular, of the limit cycles, is very important for understanding the dynamics of the differential systems. In this paper we apply the averaging theory of first order to study the limit cycles which bifurcate from the periodic orbits of the quadratic uniform isochronous center $\dot{x} = -y + xy$, $\dot{y} = x + y^2$, when this center is perturbed inside the class of all discontinuous piecewise quadratic polynomial differential systems in the plane with two pieces separated by a non-regular line. Using the Chebyshev criterion 8 is the maximum number of limit cycles which can bifurcate from the periodic orbits of this center using the averaging theory of first order, and this bound is reached. Our results for this class of discontinuous piecewise differential systems with a non-regular discontinuity line obtain two more limit cycles than for the same class of discontinuous piecewise differential systems when the discontinuity line is regular.

1. Introduction and statement of the main results

It is well-known that the second part of the *Hilbert's* $16th$ *problem* ask for an upper bound of the maximum number of limit cycles in function of the degree of the planar polynomial differential systems, and for the possible distribution of the limit cycles, see [15, 18, 20]. The possible distributions of limit cycles has been solved, see [26], but to find such upper bound remain unsolved. Therefore, more and more studies (see for example [2, 12, 20, 21] and the references therein) focus on the *weak Hilbert's* $16th$ *problem* proposed by Arnold [1], that concerns on the investigation of the maximum number of limit cycles bifurcating from the periodic orbits of the centers of polynomial differential systems when they are perturbed inside the class of all polynomial differential systems of degree n.

In this paper we study a particular case of the weak Hilbert's 16th problem, i.e. to find the maximum number of limit cycles bifurcating from the periodic orbits of a uniform isochronous center under discontinuous piecewise polynomial perturbations. Recall that for planar polynomial differential systems, Conti [9] proved that a center is called a *uniform isochronous center* if in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ it can be written in the form $\dot{r} = R(\theta, r)$, $\dot{\theta} = k$, where k is a nonzero real number.

It is easily to check that a linear differential system with a uniform isochronous center can be written as $\dot{x} = -y, \dot{y} = x$ through an affine change of variables and a rescaling of time. As indicated in [23] the quadratic polynomial differential system with a uniform isochronous

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