TOPOLOGICAL CLASSIFICATION OF SOME SD HAMILTONIAN SYSTEMS

TING CHEN[1] AND JAUME LLIBRE[2,*]

ABSTRACT. In this paper we classify the phase portraits in the Poincaré disk of the Smooth and Discontinuous (SD) Hamiltonian system with the rational Hamiltonian function $H(x,y) = y^2/2 + P(x)/Q(x,y)$, where $P(x) = a$, ax, ax^2 and $Q(x,y) = Ax^2 + By^2 + C$.

1. Introduction and statement of the main results

In this paper we deal with the Hamiltonian system

$$\dot{x} = H_y(x,y), \qquad \dot{y} = H_x(x,y), \tag{1}$$

with a rational potential

$$H(x,y) = \frac{y^2}{2} + V(x,y) = \frac{y^2}{2} + \frac{P(x)}{Q(x,y)}, \tag{2}$$

where $P(x) = a$, ax, ax^2 and $Q(x,y) = Ax^2 + By^2 + C$ with $aAB \neq 0$. The system associated to the Hamiltonian function (2) has the form

$$\dot{x} = y - \frac{P(x)\partial_y(Q(x,y))}{Q^2(x,y)}, \qquad \dot{y} = \frac{\partial_x(Q(x,y))P(x) - \partial_x(P(x))Q(x,y)}{Q^2(x,y)}, \tag{3}$$

where $\partial_x(\cdot)$ and $\partial_y(\cdot)$ indicate the derivatives of the polynomials with respect to x and y respectively, and the dot denotes derivative with respect to the real variable t, which is called the *time*. We denote by the set $L = \{(x,y)|Q(x,y) = 0\}$ the points where the Hamiltonian system (3) is not defined. To be more precise system (3) is a *Smooth* and *Discontinuous* Hamiltonian system (SD Hamiltonian system), the smooth dynamic behavior appears when the set L is empty, while the discontinuous dynamics occurs when L is not empty. By the rescaling of the time

$$\frac{dt}{d\tau} = Q^2(x,y), \tag{4}$$

the SD Hamiltonian system (3) becomes the polynomial differential system

$$\begin{aligned} x' &= yQ^2(x,y) - P(x,y)\partial_y(Q(x,y)), \\ y' &= \partial_x(Q(x,y))P(x) - \partial_x(P(x))Q(x,y), \end{aligned} \tag{5}$$

where x' and y' denote derivatives of x and y with respect to τ respectively. The new system (5) is not Hamiltonian in general, but it has a first integral of motion. For analyzing the phase portrait of the SD Hamiltonian system (3), we can study

2010 *Mathematics Subject Classification.* Primary: 34C07, 34C08.

Key words and phrases. SD Hamiltonian system, equilibrium point, infinity, separatrix, phase portrait.