ZERO–HOPF BIFURCATION IN A FAMILY OF TRITROPHIC FOOD CHAIN MODEL WITH HOLLING III–III FUNCTIONAL RESPONSE

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Abstract. In this paper we apply the second order averaging theory for obtaining an explicit expression of the small amplitude periodic solution that bifurcates from a zero-Hopf equilibrium point of a tritrophic food chain model.

This model considers logistic growth rate for the lowest trophic level, Holling functional responses type III for the middle and for the highest level.

We first prove that this model has a zero-Hopf equilibrium point, after we show that from this equilibrium bifurcates a small limit cycle, and finally we provide the explicit expression of this limit cycle.

These differential systems for which the equilibrium point is non hyperbolic are not easy to study, in particular if the equilibrium is zero-Hopf. As far as we know this is the first time that the averaging theory has been used to exhibit the analytical expression of a limit cycle which bifurcates from a zero-Hopf equilibrium in the food chain models.

1. Introduction and statements of the main results

The study of small amplitude periodic solutions bifurcating from a zero-Hopf equilibrium point of a differential system is an important topic of research in mathematical ecology. One approach for studying this problem is through the averaging theory.

A general introduction to the averaging theory is the book of Sanders, Verhulst and Murdock [17] (also you can see, [18]). In [7] Buica and Llibre extended the averaging theory for computing periodic orbits up to third order to non-smooth differential systems using the Brower degree. Later on Llibre, Novaes and Teixeira in [15] extended the averaging theory for computing periodic orbits to arbitrary order. In the Appendix A, we summarize the averaging theory up to order two that we shall need for proving the results of this paper.

In these last years the strategy that has given good results to prove the existence of limit cycles in predator-prey interaction models, was based in studying the Hopf bifurcation of the equilibrium points of such models. Thus the Hopf bifurction has been studied by Freedman and Waltman in the paper [11], where the persistence of species in a three–level food chain model. More recently Blé, Castellanos, Dela-Rosa, Loreto, Castillo-Santos and other authors have published several articles where the Hopf bifurcation for the tritrophic food chain models are analyzed considering functional responses of Holling type II, III, and IV, together with their combinations and with linear or logistic growth for the prey. In these works the existence of limit cycles that bifurcate from an equilibrium point in the positive octant is reported and their proof use the well establish theory for analyze the Hopf bifurcations, see for instance Kuznetsov [13] and the Andronov-Hopf theorem [16]. See also the papers [1, 2, 3, 4, 5, 6, 8, 9].

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