LIMIT CYCLES OF CONTINUOUS PIECEWISE SMOOTH DIFFERENTIAL SYSTEMS

JOYCE A. CASIMIRO[1] AND JAUME LLIBRE[1]

ABSTRACT. From the beginning of this century many articles have been published on the continuous and discontinuous piecewise differential systems specially in the plane. The big interest on these piecewise differential systems mainly comes from their increasing number of applications for modelling many natural phenomena. As it is usual in the planar differential systems one of the main difficulties for understanding their dynamics consists in controlling their limit cycles. The major part of papers studying continuous piecewise differential systems has an straight line as the line of separation between the differential systems forming the continuous piecewise differential systems.

In this work we consider continuous piecewise differential systems separated by a circle and formed by one linear differential center and one quadratic differential center. We study the maximum number of limit cycles that such kind of continuos piecewise differential system can exhibit.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

Around 1920's started the interest for studying the piecewise differential systems mainly in the works of Andronov, Vitt and Khaikin, see the book [17]. Nowadays this interest is increasing due to the fact that the piecewise differential systems model many processes appearing in mechanics, electronics, economy, etc., see for more details the books of Simpson [18], di Bernardo et al. [2] and, the survey of Makarenkov and Lamb [16], and the hundreds of references which appear in the references of these last citations.

The easiest continuous piecewise differential systems are the ones having only two pieces separated by a straight line in the plane $\mathbb{R}^2$ and formed by two linear differential systems. Lum and Chua in 1990 conjectured in [14, 15] that such piecewise differential systems have at most one limit cycle. We recall that a limit cycle is an isolated periodic orbit in the set of all periodic orbits of a differential system. The previous conjecture was proved in 1990 by Freire et al. [5]. Later on a distinct and shorter proof was given in 2013 by Llibre, Ordóñez and E. Ponce [11], and more recently in 2021 a new proof has been given by Carmona, Fernández-Sánchez and Novaes [3].

In the paper [10] the authors studied the discontinuous piecewise differential systems separated by a circle and formed by two linear differential systems, and proved that those systems can have at most 3 limit cycles, and that there are systems of this type having 3 limit cycles. But the same kind of piecewise differential systems being continuous on the circle has no limit cycles.

In this work before studying the limit cycles of the discontinuous piecewise differential systems separated by a circle and formed by one linear differential system and a quadratic differential system,

2010 *Mathematics Subject Classification.* 34A36, 34C07, 37G15.

Key words and phrases. Continuous piecewise differential systems, linear center, quadratic center, limit cycle.