3–DIMENSIONAL PIECEWISE LINEAR AND QUADRATIC VECTOR FIELDS WITH INVARIANT SPHERES

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Abstract. We consider the class $\mathcal{X}$ of 3-dimensional piecewise smooth vector fields that admit a first integral which leaves invariant any sphere centered at the origin. In this class, we prove that a linear vector field does not admit isolated invariant cones. Moreover, we provide the existence of at least ten 1-parameter families of crossing closed trajectories for quadratic vector fields in $\mathcal{X}$.

1. Introduction

Differential equations and dynamical systems can be used to model natural phenomena and we can obtain information about it from their solutions. An interesting tool used to understand the behavior of the solutions of a dynamical system is the existence of first integrals because, when they exist, the trajectories of the corresponding vector field remain restricted to the level surfaces of these functions. We say that a n-dimensional differential system is completely integrable when it has $n-1$ independent first integrals and the orbits of it are obtained just intersecting the level sets of the first integrals. Moreover, if it has less than $n-1$ first integrals, it is said to be partially integrable. The $2n$-dimensional Hamiltonian systems are particular cases of partially integrable systems, for which we commonly study their behavior restricted to their invariant level sets. The study of Hamiltonian systems has many applications and it is very important in mechanics, for example, as we can see in [28].

Observe that, if the system restricted to an invariant level set of the first integral has a hyperbolic closed trajectory, then the original system has a 1-parameter family of hyperbolic periodic orbits. As we will work with 3-dimensional piecewise smooth vector fields having a first integral that keeps invariant all the spheres centered at origin, in fact we deal with 1-parameter radial families. For more details about how to consider 3-dimensional smooth vector fields (resp. 3-dimensional piecewise smooth vector fields) with invariant spheres as 1-parameter radial family see for instance Section 5 of [4] (resp. [5]). In [6] it was proved that the behavior of a homogeneous vector field restricted to an invariant sphere of radius $\rho = 1$ is topologically equivalent to the behavior of the same system restricted to any other level. So, when a homogeneous vector field restricted to an invariant sphere has a limit cycle (resp. a center), the 3-dimensional vector field has an isolated (resp. non-isolated) invariant cone fulfilled of closed trajectories. On the other hand, the behavior of non-homogeneous vector fields could be totally different in distinct levels of invariant spheres (see again [6]). In this case, each hyperbolic closed trajectory restricted to an invariant sphere of radius ρ generates a 1-parameter radial family of closed trajectories of the 3-dimensional vector field near the sphere of radius ρ. So, it has locally a topological invariant

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