PHASE PORTRAITS OF (2;1) REVERSIBLE VECTOR FIELDS OF LOW CODIMENSION

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Abstract. In this paper we classify the phase portraits in the Poincaré disk of the reversible vector fields of type (2;1) of codimension 0, 1 and 2.

1. Introduction and Main Results

Given two real C^k, $k \geqslant 1$, functions of two variables P, $Q : \mathbb{R}^2 \to \mathbb{R}$ we define a *planar C^k differential system* as a system of the form

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y), \tag{1}$$

where the dot in system (1) denotes the derivative with respect to the independent variable t. We call the map $X = (P,Q)$ a *vector field*. If P and Q are polynomials then system (1) is a *planar polynomial differential system*. In this case we say that system (1) has *degree n* if the maximum of the degrees of P and Q is n. If $n = 1$ then system (1) is called a *linear differential system*. This last class of systems is already completely understood (see for instance chapter 1 of [18]). However for $n \geq 2$, that is for nonlinear differential polynomial systems we know very few things. The class of planar polynomial systems with degree $n \geq 2$ is too wide, so it is common to study more specific subclasses and to classify their topological phase portraits. See for instance [1, 8, 10, 11, 24].

In this paper we are concerned with the *reversible vector fields*. Given a C^k vector field X (not necessarily planar) and a C^k diffeomorphism $\varphi : \mathbb{R}^m \to \mathbb{R}^m$ satisfying $\varphi^2 = id_{\mathbb{R}^m}$ we say that X is φ-reversible of type $(m;s)$, for $s \in \{0, 1, \dots, m\}$, if

$$D\varphi(z)X(z) = -X(\varphi(z))$$

for all $z \in \mathbb{R}^m$ and $Fix(\varphi) = \{z \in \mathbb{R}^m : \varphi(z) = z\}$ is a s-dimensional manifold. Many types of reversible vector fields have been studied for several authors. For example in [2] all the low codimension singularities of systems (2;0)-type are classified, in [3] all (3;2)-type. In [16, 17] there is a study of the quadratic reversible vector fields of type (3;2) on the sphere $\mathbb{S}^2$.

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