STABILITY CONDITIONS FOR PIECEWISE REFRACTIVE VECTOR FIELDS PARTIALLY INTEGRABLE

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Abstract. In this article we discuss some qualitative and geometric aspects of non-smooth dynamical systems theory. Our main goal is to study stability problems inside the class of 3−dimensional piecewise smooth refractive vector fields that admit a first integral that leaves invariant any sphere centered at the origin. Global results about the stability conditions and generic one-parameter families of piecewise refractive vector fields on a two-dimensional sphere are exhibited and used to prove our main result, which establishes necessary conditions for the structural stability inside this class.

1. Introduction and statement of the main results

The theory of non-smooth dynamical systems (NSDSs) is widely used to understand the behavior of natural phenomena (see [3, 4], for example) and in 1988 Filippov gave a rigorous formulation for the problem in [5]. As in the smooth case, the integrability is an important tool that can be used to understand the behavior of the behavior of the solutions of a NSDS.

In this article we discuss some qualitative and geometric aspects of NSDS theory. Our main goal is to study stability problems inside a class of 3−dimensional partially integrable piecewise vector fields and our research program involves the study of the topological stability of pairs (ω, Z) where ω is an integrable 1-form in $\mathbb{R}^3$ and Z is a piecewise smooth vector field tangent to the foliation determined by ω. Throughout this article we assume that the leaves of ω are fixed and expressed by $\mathbb{S}^2_\lambda = \{(x, y, z) \in \mathbb{R}^3; x^2 + y^2 + z^2 = \lambda^2\}$, with λ near 0.

More specifically, we consider piecewise smooth vector fields $Z = (X, Y)$ defined on $\mathbb{R}^3$ with discontinuity set $\Sigma = \{(x, y, z); z = 0\}$, expressed as

$$Z(x,y,z) = \begin{cases} X(x,y,x), \; z \geq 0, \\ Y(x,y,x), \; z \leq 0, \end{cases} \tag{1}$$

where $X, Y : \mathbb{R}^3 \to \mathbb{R}^3$ are vector fields of C^r-class, $r \geq 2$, on $\mathbb{R}^3$ that admit $H(x, y, z) = x^2 + y^2 + z^2$ as a first integral. It means that all the spheres S^2_λ are invariant by the flow of both X and Y and then, $\mathbb{S}^2_\lambda$ are also invariant by the flow of the piecewise smooth vector field Z. We denote by $\mathcal{X}$ this class of piecewise vector fields and we endow it with the C^r- product topology.

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