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THE DISCONTINUOUS MATCHING OF TWO GLOBALLY ASYMPTOTICALLY STABLE CROSSING PIECEWISE SMOOTH SYSTEMS IN THE PLANE DO NOT PRODUCE IN GENERAL A PIECEWISE DIFFERENTIAL SYSTEM GLOBALLY ASYMPTOTICALLY STABLE

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Abstract. A differential system in the plane $\mathbb{R}^2$ is globally asymptotically stable if it has an equilibrium point p and all the other orbits of the system tend to p in forward time. In other words if the basin of attraction of p is $\mathbb{R}^2$. The problem of determining the basin of attraction of an equilibrium point is one of the main problems in the qualitative theory of differential equations. We prove that planar crossing piecewise smooth systems with two zones formed by two globally asymptotically stable differential systems sharing the same equilibrium point localized in the separation line are not necessarily globally asymptotically stable.

1. Introduction and statement of the main result

Let $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, $n \geq 2$, be a C^1 vector field. As usual, we will identify the vector field F with the ordinary differential equation

$$\dot{x} = \frac{dx}{dt} = F(x), \quad x = (x_1, \ldots, x_n), \tag{1}$$

where the dot denotes the derivative with respect to the independent variable t, called here the time.

Assume that $x^* \in \mathbb{R}^n$ is the only equilibrium point of system (1). Assume further that x^* is *locally asymptotically stable*, i.e. there exists an open neighborhood U of system x^* such that the orbits of (1) starting from U tend to x^* in forward time. The *basin of attraction* of x^* is the largest open set whose elements satisfy the above condition. Of course, the neighborhood U is contained in the basin of attraction of x^*. The equilibrium point x^* or the vector field F is *globally asymptotically stable* if its basin of attraction is the whole $\mathbb{R}^n$.

The problem of determining the basin of attraction of an equilibrium point of a vector field is of great importance for applications of the stability theory of ordinary differential equations. The globally asymptotically stable problem is closely related to the Markus-Yamabe Conjecture [9].

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1991 *Mathematics Subject Classification.* Primary: 34D23, 34C07.

Key words and phrases. Piecewise differential system, global asymptotic stability, limit cycle.