A WEAKENED MARKUS-YAMABE CONDITION FOR GENERALIZED φ-LAPLACIAN LIÉNARD SYSTEMS

LUIS BARREIRA[1], JAUME LLIBRE[2] AND CLAUDIA VALLS[3]

Abstract. For a general autonomous planar polynomial differential system it is difficult to find conditions that are easily verifiable and which guarantee global asymptotic stability. In this paper we study when the following three conditions

(c1) the divergence of the differential system is negative in the whole $\mathbb{R}^2$;

(c2) the differential system has a unique equilibrium point $p \in \mathbb{R}^2$; and

(c3) the equilibrium point p is locally asymptotically stable;

imply for the generalized φ-Laplacian Liénard systems

$$x' = y, \quad y' = -g(x) - f(x)y(y^2 - 3\alpha y + 3(\alpha^2 + \beta^2)), \quad (\alpha, \beta) \in \mathbb{R}^2 \setminus \{(0,0)\},$$

the global asymptotic stability of the equilibrium point p. Here $g(x)$ and $f(x)$ are polynomials of degree n and m, respectively. In particular we prove that if $n \leq m+3$ with $n \neq m+1$, then the generalized φ-Laplacian Liénard systems satisfying conditions (c1)–(c3) are globally asymptotically stable. We note that these three conditions are weaker than the Markus-Yamabe conditions.

1. Introduction and statement of the main results

Liapunov's approach is probably the most general method used to find conditions which guarantee the global asymptotic stability of an equilibrium point in a differential system in any dimension. For the 2–dimensional autonomous system

$$x' = P(x, y), \quad y' = Q(x, y), \tag{1}$$

with $X = (P, Q)\colon \mathbb{R}^2 \to \mathbb{R}^2$, some authors looked for easily verifiable conditions in the function X which may give global asymptotic stability. A first step towards this direction is the so-called Markus–Yamabe conjecture in two dimensions (see [4, 5, 6]) which was proven in 1993. This result shows that global asymptotic stability is obtained if there is only one equilibrium point and the eigenvalues of the Jacobian matrix $DX(x, y)$ have negative real part for all $(x, y) \in \mathbb{R}^2$ (or that the trace of DX is negative and the determinant of DX is positive for all $(x, y) \in \mathbb{R}^2$). However these conditions are not easy to verify and the aim of several later works has been to weaken the Markus-Yamabe condition and still obtain global asymptotic stability for some classes of differential systems (1) in dimension 2.

Some researchers have tried to weaken the Markus–Yamabe condition by dropping the determinant condition and replacing it by other conditions. In this context, guided in particular by results in [2] for polynomial differential systems of degrees 2 and 7 in the plane (we recall that the degree of a polynomial vector field X is n if the

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