ON THE NUMBER OF LIMIT CYCLES FOR TWO CLASSES OF POLYNOMIAL DIFFERENTIAL SYSTEMS

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ABSTRACT. This paper deals with two classes of polynomial differential systems which are generalized rigid systems, i.e. differential systems whose orbits rotate with constant angular velocity. We give the bounds for the maximum number of limit cycles of such polynomial differential systems provided by the averaging theory of first order. These bounds are reached.

1. Introduction and statement of the main results

In the qualitative theory of planar polynomial differential systems, one of the most important problems is the determination and distribution of their limit cycles. This problem is known as the famous Hilbert's 16th problem [11, 14]. Since this problem is very difficult and it remains open, many mathematicians pay attention to particular classes of planar polynomial differential systems, as the Liénard systems see [4, 7], or the Z_2-equivariant systems see [5, 13], ...

A *rigid polynomial differential system* can be written as

$$\frac{dx}{dt} = -y + xF(x,y), \qquad \frac{dy}{dt} = x + yF(x,y), \tag{1}$$

where $F(x,y)$ is polynomial with $F(0,0) = 0$. Using the polar coordinates (r,θ) defined by $x = r\cos\theta$, $y = r\sin\theta$, the rigid system (1) becomes

$$\frac{dr}{dt} = rF(r\cos\theta, r\sin\theta), \qquad \frac{d\theta}{dt} = 1.$$

It is worth to note that when the origin $(0,0)$ of this differential system is a center, this class of rigid systems have been studied extensively, see for instance [8, 9, 10, 15, 22].

In this paper we deal with the polynomial differential systems

$$\frac{dx}{dt} = -y\left(x^2+y^2\right)^{k/2} + xU_l(x,y), \qquad \frac{dy}{dt} = x\left(x^2+y^2\right)^{k/2} + yU_l(x,y), \tag{2}$$

where k is an even positive integer, and $U_l(x,y) = \sum\limits_{i+j=l} a_{i,j}x^iy^j$ is a homogeneous polynomial of degree l in the variables x and y.

In polar coordinates the differential system (2) writes

$$\frac{dr}{dt} = r^{l+1}U_l(\cos\theta, \sin\theta), \qquad \frac{d\theta}{dt} = r^k. \tag{3}$$

Doing the time scaling $d\tau = r^k dt$ the differential system (3) becomes the rigid system

$$\frac{dr}{d\tau} = r^{l+1-k}U_l(\cos\theta, \sin\theta), \qquad \frac{d\theta}{d\tau} = 1.$$

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2010 *Mathematics Subject Classification.* Primary34C37, 34C07, 37G15.
Key words and phrases. Averaging theory; limit cycle, rigid differential systems.