DYNAMICS OF THE STATIC STAR DIFFERENTIAL SYSTEM FROM THE MATHEMATICAL AND PHYSICAL POINT OF VIEWS

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Abstract. We classify all the topologically non-equivalent phase portraits of the quadratic polynomial differential system

$$\dot{x} = (1-2x)(y-x), \quad \dot{y} = y\left(2 - \gamma y - \frac{5\gamma-4}{\gamma-1}x\right),$$

in the Poincaré disc for all the values of the parameter $\gamma \in \mathbb{R} \setminus \{1\}$.

The differential system

$$\frac{dx}{dt} = y - x, \quad \frac{dy}{dt} = \frac{y}{1-2x}\left(2 - \gamma y - \frac{5\gamma-4}{\gamma-1}x\right),$$

when the parameter $\gamma \in (1, 2]$ models the structure equations of a static star in general relativity in the case of the existence of a homologous family of solutions, being $x = m(r)/r$ where $m(r) \geq 0$ is the mass inside the sphere of radius r of the star, $y = 4\pi r^2\rho$ where ρ is the density of the star, and $t = \ln(r/R)$ where R is the radius of the star. We classify the possible values of $m(r)/r$ and $4\pi r^2\rho$ when $r \to 0$.

1. Introduction and the main results

The structure equations of a static star in general relativity in the case of the existence of a homologous family of solutions are

$$\begin{aligned} \dot{x} &= y - x, \\ \dot{y} &= \frac{y}{1-2x}\left(2 - \gamma y - \frac{5\gamma-4}{\gamma-1}x\right), \end{aligned} \tag{1}$$

where the parameter γ varies in the interval $(1, 2]$, and the dot denotes derivative with respect to the variable $t = \ln(r/R)$ being R the radius of the star. Therefore, from the physical point of view we are interested in the solutions defined in the interval $t \in (-\infty, 0)$. Here $x = m(r)/r$ where $m(r) \geq 0$ is the mass inside the sphere of radius r of the star, $y = 4\pi r^2\rho$ being ρ the density of the star. For more details on the differential system (1) see [3, 4, 5, 7, 8, 9].

We remark that from the physical point of view and since $x > 0$ and $y > 0$ we are mainly interested in the dynamics of the differential system (1) with $\gamma \in (1, 2]$ in the set Q^* formed by the positive quadrant $Q = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0\}$ of $\mathbb{R}^2$ without the straight line $x = 1/2$ where the differential system (1) is not defined.

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