GLOBAL CENTERS OF A FAMILY OF CUBIC SYSTEMS

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Abstract. Consider the family of polynomial differential systems of degree 3, or simply cubic systems

$$x' = y, \quad y' = -x + a_1x^2 + a_2xy + a_3y^2 + a_4x^3 + a_5x^2y + a_6xy^2 + a_7y^3,$$

in the plane $\mathbb{R}^2$.

An equilibrium point (x_0, y_0) of a planar differential system is a *center* if there is a neighborhood $\mathcal{U}$ of (x_0, y_0) such that $\mathcal{U}\setminus\{(x_0, y_0)\}$ is filled with periodic orbits. When $\mathbb{R}^2 \setminus \{(x_0, y_0)\}$ is filled with periodic orbits, then the center is a *global center*.

For this family of cubic systems Lloyd and Pearson characterized in [12] when the origin of coordinates is a center. We classify which of these centers are global centers.

1. Introduction and statement of the main result

Let $P, Q : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be polynomials and consider the differential system

$$x' = P(x, y), \qquad y' = Q(x, y). \tag{1}$$

Denote by $X(x, y) = (P(x, y), Q(x, y))$ the vector field associated to the differential system (1). The *degree* d of system (1) is the maximum of the degrees of the polynomials P and Q. Here the apostrophe denotes derivative with respect to the time t. A point (x_0, y_0) is an *equilibrium point* of system (1) if $X(x_0, y_0) = (0, 0)$.

An equilibrium point (x_0, y_0) of system (1) is a *center* if there is a simply connected open neighborhood W of (x_0, y_0) such that (x_0, y_0) is the only equilibrium point in W and all the trajectories contained in $W\setminus\{(x_0, y_0)\}$ are periodic. The largest simply connected open neighborhood $\mathcal{P}$ of (x_0, y_0) such that $\mathcal{P}\setminus\{(x_0, y_0)\}$ is filled of periodic trajectories is called the *period annulus*. When $\mathcal{P} = \mathbb{R}^2$ the point (x_0, y_0) is a *global center*.

Dulac [5] and Poincaré [13] were the first in studying the centers of the differential systems in the plane. While Conti [4] was the first in studying the global centers.

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