ENTROPY STABILITY AND MILNOR-THURSTON INVARIANTS FOR BOWEN-SERIES-LIKE MAPS

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Abstract. We define a family of discontinuous maps on the circle, called Bowen-Series-like maps, for geometric presentations of surface groups. The family has $2N$ parameters, where $2N$ is the number of generators of the presentation. We prove that all maps in the family have the same topological entropy, which coincides with the volume entropy of the group presentation. This approach allows a simple algorithmic computation of the volume entropy from the presentation only, using the Milnor-Thurston theory for one dimensional maps.

1. Introduction

Let Σ be a closed compact surface of rank larger than 2 and let $P = \langle X|R\rangle$ be a presentation of its fundamental group $G := \pi_1(\Sigma)$. Since the rank is larger than 2, the surface Σ is hyperbolic in the geometrical sense, G is a hyperbolic group in the sense of Gromov [16, 14] and its boundary ∂G is homeomorphic to the circle S^1. We consider the Cayley graph of the group presentation $\mathrm{Cay}^1(G,P)$ and the Cayley 2-complex $\mathrm{Cay}^2(G,P)$. The presentation P is called *geometric* if $\mathrm{Cay}^2(G,P)$ is homeomorphic to a plane. In particular, $\mathrm{Cay}^1(G,P)$ is a planar graph. This property is satisfied by the classical presentations of any surface group (see for instance [22]).

For a hyperbolic group G with a presentation $P = \langle X|R\rangle$, let $\mathscr{X}$ be the *generating set*, defined as the set of generators of the presentation and their inverses (by abuse of language, sometimes we will use the term *generator* to refer to any element of $\mathscr{X}$). For any $g \in G$, the *length of* g, denoted by $\mathrm{length}(g)$, is the number of symbols of a minimal word in the alphabet $\mathscr{X}$ representing g. It coincides with the number of edges in any geodesic segment (shortest path) in the Cayley graph $\mathrm{Cay}^1(G,P)$ connecting the identity element of G to the vertex g. The *growth function*

$$\sigma_m := \mathrm{Card}\,\{g \in G : \ \mathrm{length}(g) = m\}, \tag{1}$$

which is also the number of vertices at distance m from the identity in the Cayley graph, plays a central role in geometric group theory [11, 13, 15]. Its exponential growth rate is called the *volume entropy*, defined as

$$h_{\mathrm{vol}}(G,P) = \lim_{m\to\infty} \frac{1}{m}\log(\sigma_m). \tag{2}$$

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