TOPOLOGICAL ENTROPY, SETS OF PERIODS AND TRANSITIVITY FOR GRAPH MAPS

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ABSTRACT. Transitivity, the existence of periodic points and positive topological entropy can be used to characterize complexity in dynamical systems. It is known that for graphs that are not trees, for every $\varepsilon > 0$, there exist (complicate) totally transitive maps (then with cofinite set of periods) such that the topological entropy is smaller than ε (simplicity). We show by means of three examples that for any graph that is not a tree, relatively simple maps (with small entropy) which are totally transitive (and hence robustly complicate) can be constructed so that the set of periods is also relatively simple. To numerically measure the complexity of the set of periods we introduce a notion of a *boundary of cofiniteness*. Larger boundary of cofiniteness means simpler set of periods. With the help of the notion of boundary of cofiniteness we can state precisely what do we mean by extending the entropy simplicity result to the set of periods: *there exist relatively simple maps such that the boundary of cofiniteness is arbitrarily large (simplicity) which are totally transitive (and hence robustly complicate)*. Moreover, we will show that, the above statement holds for arbitrary continuous degree one circle maps.

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