- the birth rate is denoted by $B(N)$;
- the rate of the removal members at which the infective members go into the removed class is denoted by ν;
- the probability that the individuals of the removed class loss their immunity and go into the susceptible class is r_1.

This model assumes that all the new born members are all susceptible (see [15]).

Differential system (1) is reduced in [15] to the two-dimensional differential system

$$\frac{\mathrm{d}I}{\mathrm{d}\tau} = IH(I, N_0 - I - R) - (r_2 + \nu)I, \quad \frac{\mathrm{d}R}{\mathrm{d}\tau} = \nu I - (r_2 + r_1)R. \tag{2}$$

A more special model has been brought forward in [13,16–18] by taking $H(I,S) = KIS$ and $r = \nu/(r_2 + \nu)$, $h = \nu/(r_2 + r_1)$, $a = K/(r_2 + \nu)$, $t = (r_2 + \nu)\tau$, with $K > 0$. Then model (2) becomes

$$\frac{\mathrm{d}I}{\mathrm{d}t} = aI^2(N_0 - I - R) - I, \quad \frac{\mathrm{d}R}{\mathrm{d}t} = r(I - \frac{R}{h}). \tag{3}$$

To decrease the number of parameters of system (3), we rescale its variables taking $I = \alpha x$, $R = \beta y$, $T = \gamma t$. Therefore system (3) writes

$$\dot{x} = a\alpha\gamma N x^2 - a\alpha^2\gamma x^3 - a\alpha\beta\gamma x^2 y - \gamma x, \quad \dot{y} = \frac{\alpha\gamma r}{\beta}x - \frac{\gamma r}{h}y. \tag{4}$$

Choosing $\alpha = \frac{r}{ahN}$, $\beta = \frac{r}{aN}$, $\gamma = \frac{h}{r}$, system (4) becomes

$$\dot{x} = -\frac{r}{ahN^2}x^3 - \frac{r}{aN^2}x^2 y + x^2 - \frac{h}{r}x, \quad \dot{y} = x - y. \tag{5}$$

Changing the names of the parameters as $b = h/r$, $c = r/(ahN^2)$ and $d = r/(aN^2)$, system (5) only depends on three parameters and it writes

$$\dot{x} = -bx - cx^3 - dx^2y + x^2, \quad \dot{y} = x - y, \tag{6}$$

where b, c and d are positive parameters.

The objective of this paper is to study the phase portraits in the Poincaré disc of system of system (6), and in particular determine its attractors, the important objects in biology.

System (6) is defined in the plane $\mathbb{R}^2$, but to control its orbits which escape or come from infinity we extend it to the Poincaré disc. Roughly speaking the Poincaré disc $\mathbb{D}$ is the closed unit disc centered at the origin of coordinates of $\mathbb{R}^2$, the interior of $\mathbb{D}$ is identified with $\mathbb{R}^2$ and its boundary, the circle $\mathbb{S}^1$ is identified with the infinity of $\mathbb{R}^2$. Note that in the plane $\mathbb{R}^2$, we can go to infinity in as many directions as points have the circle. For more details on the Poincaré disc, see Chapter 5 of [19].

Our main result is the following.

Theorem 1.1: *The phase portraits in the Poincaré disc of systems* (6) *are topologically equivalent to one of the phase portraits given in Figure 1.*

Theorem 1.1 is proved in Section 4 assuming that the following conjecture holds. At this moment, we only have numerical evidence that it must hold. It is well known that in general to prove the existence and uniqueness of a limit cycle is a very difficult problem.

Conjecture. *The differential system* (6) *has at most one limit cycle.*

A periodic orbit of system (6) isolated in the set of all periodic orbits of the system is a *limit cycle*.