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ORIGINAL RESEARCH

Phase Portraits of a Class of Cubic Systems with an Ellipse and a Straight Line as Invariant Algebraic Curves

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Abstract
In this paper we classify the phase portraits in the Poincaré disc of a class of cubic polynomial differential systems having an invariant ellipse and an invariant straight line. We prove that such a class of cubic polynomial differential systems have exactly 43 topologically different phase portraits in the Poincaré disc. Also we obtain that the invariant ellipse in two of these phase portraits is a limit cycle.

Keywords Phase portraits in the Poincaré disc · Limit cycle · Cubic polynomial differential system · Invariant ellipse · invariant straight line

Mathematics Subject Classification 37G15

Introduction and Statements of the Main Results

The *phase portrait* of a differential system defined in the plane $\mathbb{R}^2$ consists in describing $\mathbb{R}^2$ as union of all the orbits of the differential system. The phase portrait of a differential system provides the maximal qualitative information about its dynamics. This is the best information which can be given for a differential system whose orbits cannot be given explicitly in function of the time.

A *polynomial differential system* in the plane $\mathbb{R}^2$ is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

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