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Solar System transport in a chain of bicircular models

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Abstract

The bicircular problem (BCP) is a simplified model for the four body problem. In this model, we assume that the Sun and Jupiter are revolving in circular orbits around their center of mass, and a planet moves in a circular orbit around their barycenter. This is not a coherent model in the sense that the trajectories of the Sun, Jupiter and the planet do not satisfy Newton's equations. In a first step, our aim is to consider the Solar System as a set of coupled bicircular models. This is done in order to obtain a first insight of transport in the Solar System that may be explained using the *separated* bicircular problems. The invariant manifolds of convenient periodic orbits of each particular bicircular problem are considered in order to find connections between two consecutive problems. These connections allow to obtain a mechanism to explain transport of infinitesimal particles towards the inner Solar System. Finally the results obtained should be validated using the complete Solar System.

1. Introduction. Motivation

Comets, asteroids and small particles in the solar system are capable of performing transfers from their original location to very distant places: **natural transport**.

Goal: Give a dynamical mechanism to explain transport in the Solar System.

Preliminaries: Consider a chain of Restricted Three Body Problems, Sun+Planet, and the invariant manifolds associated to the central manifold of the collinear points L_1 and L_2 of each RTBP.

- Invariant manifolds of L_1 and L_2: orbital elements

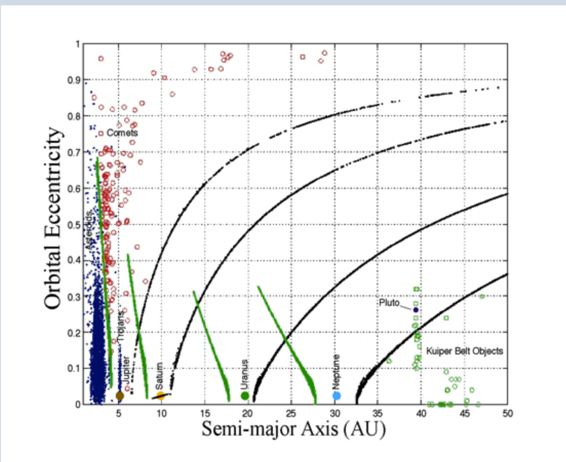

Figure : Keplerian elements a, e of the manifolds of L_1 and L_2 (Lo, Ross: personal comunication)

- Invariant manifolds of L_1 and L_2: the minimum and maximum heliocentric distances.

PCR3BP	$R\ W^{u(s)}_{L_1}$		$R\ W^{u(s)}_{L_2}$	
	min	max	min	max
Sun-Mercury	0.37547	0.38562	0.38858	0.39918
Sun-Venus	0.67152	0.71658	0.73011	0.78024
Sun-Earth	0.92328	0.98998	1.01008	1.08488
Sun-Mars	1.46684	1.51644	1.53094	1.58331
Sun-Jupiter	3.02493	4.85550	5.56589	9.46402*
Sun-Saturn	6.63467*	9.12494	9.99818	14.04464
Sun-Uranus	15.87518	18.75222	19.69233	23.49574*
Sun-Neptune	23.35734*	29.33805	30.89615	37.25573

Figure : Minimum and maximum distances of the invariant manifolds to the origin. Asterisks indicate the consecutive Sun-planet systems with short-time transfer ([2])

- Invariant manifolds of Lyapunov periodic orbits around L_1 and L_2

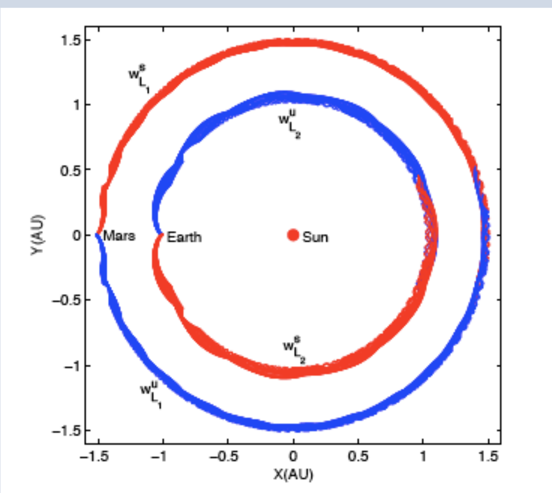

Figure : Branches of W^u and W^s associated to L_1 and L_2 in the restricted Sun+Mars and Sun+Earth problems respectively [2]

Methodology:

- Consider a chain of Bicircular Problems (BCP) with the Sun, Jupiter, a different planet and a massless particle
- Compute the invariant manifolds associated to the dynamical substitutes of the collinear equilibrium points
- Look for intersections of invariant manifolds
- Comparison of results with the real Solar Sytem

2. The bicircular model

The bicircular problem (BCP) is a simplified model for the four body problem. In this model where the Sun and Jupiter are revolving in circular orbits around their center of mass, and a planet moves in a circular orbit around this baricenter (**Remark: it is not coherent**).

Hamiltonian: Using synodical (rotating) coordinates of the Sun–Jupiter system, with the Sun and Jupiter at fixed positions $(\mu,0)$ and $(\mu-1,0)$ the Hamiltonian becomes ([1])

$$H = \underbrace{\frac{1}{2}(p_x^2+p_y^2+p_z^2)+yp_x-xp_y-\frac{1-\mu}{\rho_1}-\frac{\mu}{\rho_2}}_{\text{RTBP Sun+Jupiter}}$$
$$\underbrace{-\frac{\mu_P}{\rho_P}-\frac{\mu_P}{a_P^2}(y\sin\theta-x\cos\theta)}_{\text{planet perturbation}}$$

where

$$\begin{aligned}
\rho_1 &= ((x-\mu)^2+y^2+z^2)^{1/2},\\
\rho_2 &= ((x-\mu+1)^2+y^2+z^2)^{1/2},\\
\rho_P &= ((x-a_P\cos\theta)^2+(y+a_P\sin\theta)^2+z^2)^{1/2},\\
\theta &= \theta_0+t(1-\omega_P),\\
\mu &= \frac{m_J}{m_J+m_S}, \qquad \mu_P=\frac{m_P}{m_J+m_S}
\end{aligned}$$

m_S, m_J, m_P are the masses of the Sun, Jupiter and the planet, θ_0 and ω_P are the initial phase and rotational frequency of the planet.

3. Dynamical substitutes of equilibrium points

The BCP may be regarded as a periodic perturbation of the restricted three-body problem. The collinear points L_i, $i=1,2,3$ give rise to periodic orbits in the BCP and inherit their instability.

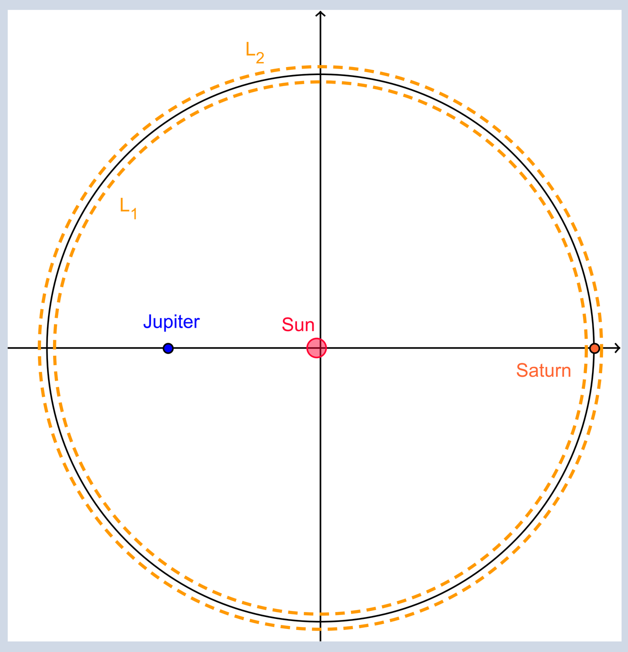

Figure : Bicircular model Sun, Jupiter and Saturn, and the dynamical substitues of L_1 and L_2

Parametrization:

- of a PO of period $T=2\pi/\omega$:
$$\varphi(\theta) := \phi^{\theta_0}_{(\theta-\theta_0)/\omega}(x_0).$$
- an invariant manifold of a PO:
$$\bar{\psi}(\theta,\xi) := \varphi(\theta)+\xi v(\theta).$$
with
$$v(\theta) = \Lambda^{-\frac{\theta-\theta_0}{2\pi}} D\phi^{\theta_0}_{(\theta-\theta_0)/\omega}(x_0)v_0.$$

4. Some results

- For transport towards inside the Solar System, the invariant manifolds that should play a role are $W^u(L_1)$ and $W^s(L_2)$ of each bicircular problem.
- We look for intersections between $W^u(OPL_1)$ (for Sun-Jupiter-Neptune BCP system) and $W^s(OPL_2)$ (for Sun-Jupiter-Uranus BCP one).

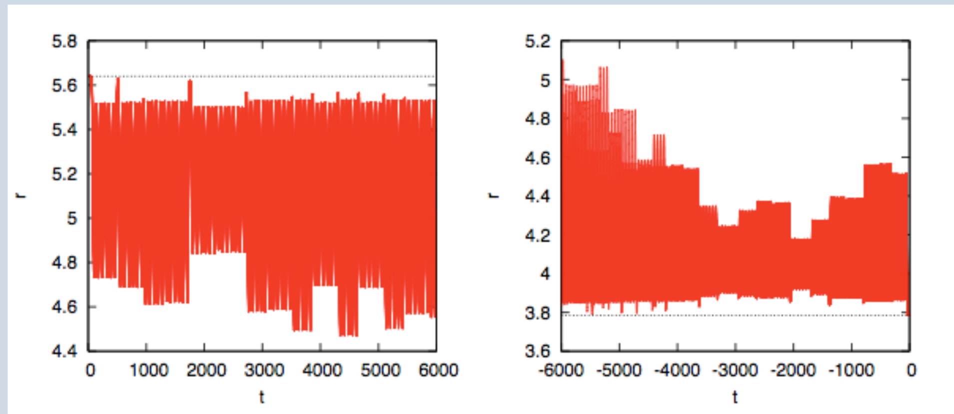

Figure : Function $r(t)$ for $W^u(OPL_1)$ of the Sun-Jupiter-Neptune system (left) and $W^s(OPL_2)$ of the Sun-Jupiter-Uranus system.

Bicircular problem	$r(W^u(L_1))$ min	$r(W^s(L_2))$ max
S-J-Neptune	4.09629	
S-J-Uranus	2.40607	5.57242
S-J-Saturn	0.03296	>10

Table : Minimum and maximum values of $r(t)$ for $W^u(L1)$ and $W^s(L_2)$ (resp) and $|t|\le 10^4$

- Preliminary results in the Solar System
  - Initial conditions: Close to the orbit of the L1/L2 point of the Sun–Neptune system
  - Six sets, of 10 particles each, uniformly distributed along the orbit of the L2 point of the Sun–Neptune system.
  - Total time integration: 10^5 years.

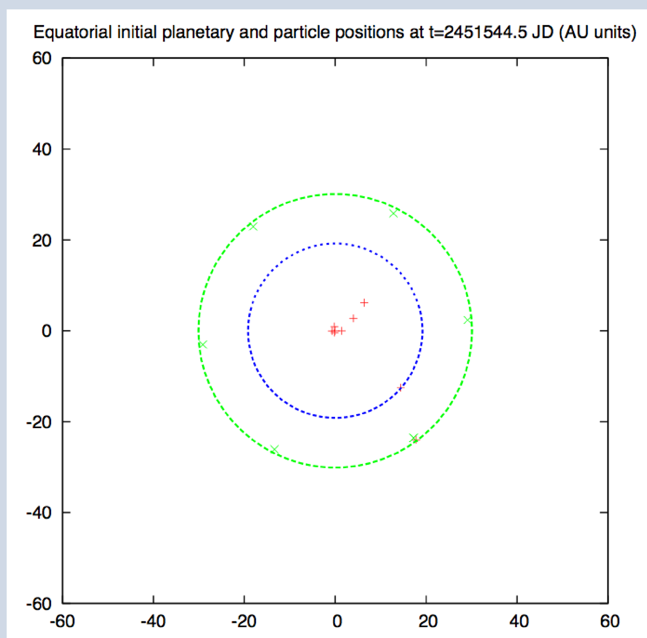

Figure : Real Solar System and some particles close to the dynamical substitute OPL_2 of Sun-Neptune.

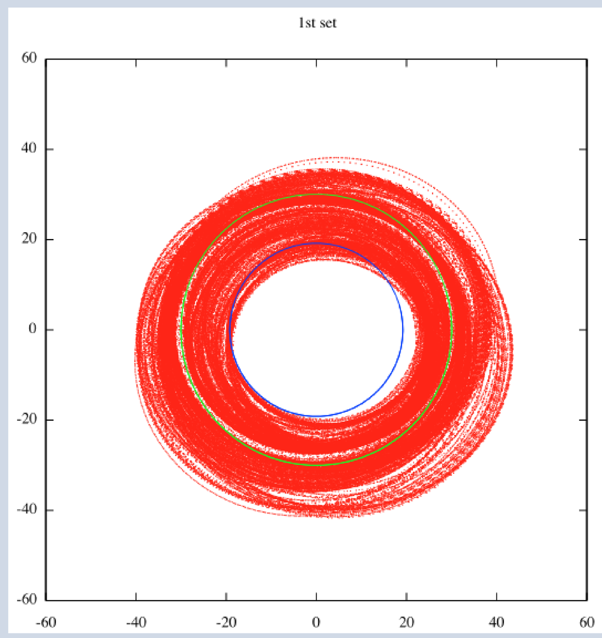

Figure : Orbits described by the particles in (x,y).

Conclusions and work in progress

- The behaviour of the manifolds of the Lyapunov periodic orbits in a chain of BCP gives a first indicator of transport in the Solar System
- Some numerical simulations in the real Solar Sytem remain to be done to check accordance with the previous results.

References

[1] G. Gómez, J. Llibre, R. Martínez, C. Simó. Study on orbits near the triangular libration points in the perturbed restricted three-body problem. *Report ESOC 6139/84/D/JS(SC)*, 1987.

[2] Y. Ren, J.J. Masdemont, G. Gómez, E. Fantino. On the mechanisms of natural transport in the Solar System. Preprint, (2011)