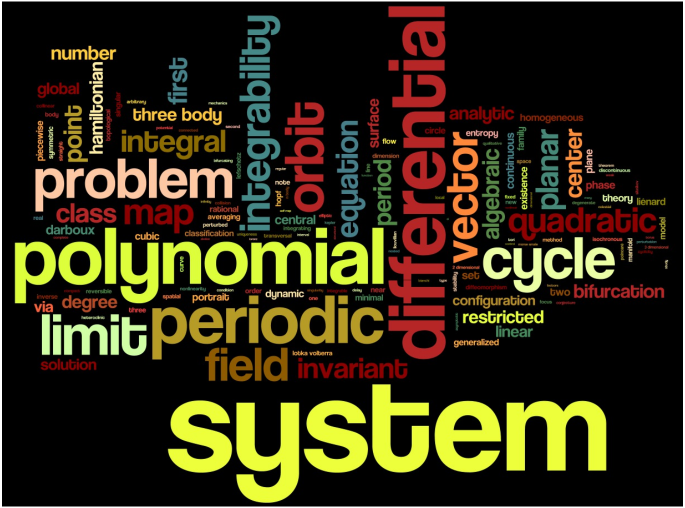
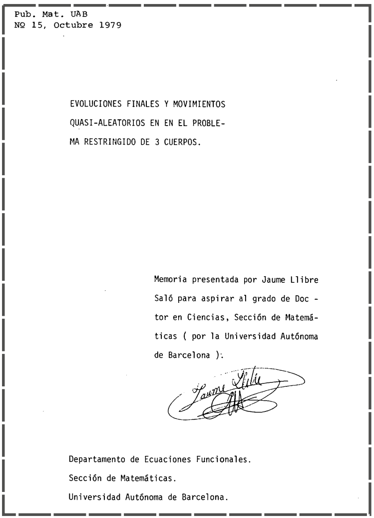
Jaume's contributions to mathematics

Title’s words in Jaume’s work



The origin

- J. Llibre (Advisor: C. Simó)
  Evoluciones finales y movimientos quasi-aleatorios en el problema restringido de 3 cuerpos
  Publ. Sec. Mat. Univ. Autònoma Barcelona, 15, 1-124, 1979.



Pub. Mat. UAB
Nº 15, Octubre 1979

EVOLUCIONES FINALES Y MOVIMIENTOS QUASI-ALEATORIOS EN EN EL PROBLEMA RESTRINGIDO DE 3 CUERPOS.

Memoria presentada por Jaume Llibre Saló para aspirar al grado de Doctor en Ciencias, Sección de Matemáticas (por la Universidad Autónoma de Barcelona).

Departamento de Ecuaciones Funcionales.
Sección de Matemáticas.
Universidad Autónoma de Barcelona.

The problem's framework

J. Chazy
Sur l'allure finale du mouvement dans le problème des trois corps I,II,III.
Ann. Ecole Norm. Sup. 39, (1922) 29-130 J. Math. Pures Appl. 8, (1929) 353-380, Bull Astron. 8, (1932) 403-436.

K. A. Sitnikov
The existence of oscillatory motions in the three body problem
Soviet Phys. Dokl. 5, (1961) 647-650.

V. M. Alekseev
Quasirandom dynamical systems, I, II, III.
Math USSR-Sb. 5 (1968), 73-128; 6, 505-560; 7 (1969), 1-43.

J. Moser
Stable and random motion in dynamical systems
Princeton: University Press (1973).

The Main result

Theorem

For small values of the mass parameter and Jacobi constant C large enough, for every sequence of integers $\{a_n\}$ with a_n sufficiently large, there exists an orbit such that the successive number of revolutions given by the primaries m_1, m_2 between two consecutive passes of the small body through a fixed ray (through the origin in the sidereal system) L, take the values $\{a_n\}$ prescribed by the sequence.

Publications

- J. Llibre and C. Simó
  Oscillatory solutions in the planar restricted three-body problem
  Math. Ann., 248 (1980) 153-184.
- J. Llibre and C. Simó
  Some homoclinic phenomena in the three-body problem
  JDE, 37 (1980) 444-465.

Math. Ann. 248, 153–184 (1980)

Mathematische Annalen

Oscillatory Solutions in the Planar Restricted Three-Body Problem

Jaume Llibre[1] and Carles Simó[2]

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2 Facultat de Matemàtiques, Universitat de Barcelona, Spain

1. Introduction

In this paper we prove the existence of oscillatory solutions in the circular planar restricted three-body problem. Let m_1, m_2 be the masses of the primaries normalized in such a way that $m_1=1-m$, $m_2=m$, $m\in(0,1)$. Units of length and time are chosen in order to have one unit of distance between the primaries and a mean motion equal to 1.

For the position of the infinitesimal body m_3 we use both the sidereal coordinates (X,Y) and the synodical ones (x,y). In the last system the two primaries are fixed at $(m,0)$, $(m-1,0)$, respectively. Then the equations of motion are (see Szebehely [7]):

$$\ddot{x}-2\dot{y}=\frac{\partial\Omega}{\partial x},\qquad \ddot{y}+2\dot{x}=\frac{\partial\Omega}{\partial y},\tag{1.1}$$

where $\Omega(x,y)=(x^2+y^2)/2+(1-m)/r_1+m/r_2+m(1-m)/2$, and $r_1^2=(x-m)^2+y^2$, $r_2^2=(x+1-m)^2+y^2$.

System (1.1) has the Jacobi first integral

$$C=C(x,y,\dot{x},\dot{y})=2\Omega(x,y)-(\dot{x}^2+\dot{y}^2)\tag{1.2}$$

which equals two times the difference between the angular momentum with respect to the origin, M, and the energy, h, both in sidereal coordinates.

For large values of C the zero velocity curves defined by $2\Omega(x,y)-C=0$ have three components. We only consider motion in the unbounded component R of the admissible region $2\Omega(x,y)\geqq C$. Let r be the distance from m_3 to the origin. An oscillatory solution is characterized by the fact that $\limsup_{t\to+\infty} r(t)=+\infty$, but $\liminf_{t\to+\infty} r(t)<+\infty$. We prove the existence of such orbits by the usual method of symbolic dynamics. Furthermore we prove the existence of all possible types of final evolution.

0025-5831/80/0248/0135/$06.40

Jaume's lines of research post PhD

- Celestial mechanics
  - Restricted 3-body problems
  - Central configurations
  - Periodic orbits
- Discrete Dynamical Systems
  - Periods and Combinatorial Dynamics for trees and graphs
  - Periods and Leftschetz numbers on surfaces
- Qualitative theory of differential equations on the plane
  - Qualitative theory of ODE and Hilbert 16th problem
  - Darboux theory and Integrability
  - Non-smooth ODE

Discrete Dynamical Systems. Motivation

- A. N. Sharkovsky
  Co-existence of the cycles of a continuous mapping of the line into itself
  Ukrain. Math. Z., 16 (1964), 61-71.

- M. Misiurewicz
  Periodic points of maps of degree one of a circle.
  ETDS 2 (1982), 221-227.

Qualitative theory of planar ODE's. Motivation

W. A. Coppel
A survey of quadratic systems
JDE 2 (1966) 293-304.

G. Darboux
Mémoire sur les équations différentielles algébriques du premier ordre et du premier degré (Mélanges)
Bull. Sci. Math. 2ème Série 2 (1878), 60-96, 123-144, 151-200.

Y. Yan Qian (and others)
Theory of limit cycles
Translations Math. Monograph 66 (1986).

Working together: Coauthors

JAUME, up to these days (but this is a dynamical system... far away from an equilibrium point), has about

215 coauthors

Well... perhaps not so nicely organized....

Well... perhaps not so nicely organized....

An extraordinary record: PhD Students

D. L. Maranhão and J. Llibre
Ejection-collision orbits and invariant punctured tori in a restricted four-body problem
Celestial Mech. Dynam. Astronom, 71 (1998), 1-14.

Ll. Alsedà, D. Juher and **P. Mumbrú**
On the preservation of combinatorial types for maps on trees
Ann. Inst. Fourier (Grenoble), 55 (2005), 2375-2398.

An extraordinary record

E. Lacomba and J. Llibre
Dynamics of a galactic Hamiltonian system
J. Math. Phys., 53 (2012).

Ll. Alsedà, F. Mañosas and **W. Szlenk**
A characterization of the uniquely ergodic endomorphisms of the circle
Proc. Amer. Math. Soc., 117 (1993), 711-714.

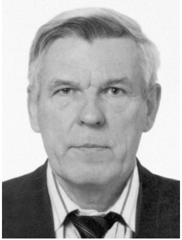
An extraordinary record

- **J. Chavarriga** and J. Llibre
  Invariant algebraic curves and rational first integrals for planar polynomial vector fields
  JDE, 169 (2001), 1-16.

- M. Cobo, **C. Gutierrez** and J. Llibre
  On the injectivity of $\mathcal{C}^1$ maps of the real plane
  Canad. J. Math., 54 (2002), 1187-1201.

An extraordinary record

L. A. Cherkas, J. C. Artés and J. Llibre
Quadratic systems with limit cycles of normal size
Bul. Acad. Repub. Mold. Mat., 41 (2003), 31-46.



Jaume's contributions on Celestial Mechanics

(brief summary)

Restricted three-body problem

J. Llibre
On the restricted three-body problem when the mass parameter is small
Celestial Mechanics, 28 (1982), 83-105.

J. Llibre, R. Martínez and C. Simó
Tranversality of the invariant manifolds associated to the Lyapunov family of periodic orbits near in the restricted three-body problem
JDE, 58 (1985), 104-156.

G. Gómez, J. Llibre, R. Martínez and C. Simó
Station keeping on libration point orbits
ESA contract, No. 5648/83/D/JS/SC, 1983-1985.

Mexican connection

M. Alvarez and J. Llibre
Heteroclinic orbits and Bernoulli shift for the elliptic collision three–body problem
Arch. Ration. Mech. Anal. , 156 (2001), 317-357.

J. Bernat, J. Llibre and E. Pérez-Chavela
On the planar central configurations of the 4-body problem with three equal masses
Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. 16 (2009), 1-13.

S. R. Kaplan, E. A. Lacomba and J. Llibre
Symbolic dynamics of the elliptic rectilinear restricted 3-body problem
Discrete Contin. Dyn. Syst. Ser. S, 1 (2008), 541-555.

Central Configurations

- J. Llibre
  On the number of central configurations in the N–body problem
  Celestial Mechanics 50 (1991), 89-96.
- J. M. Cors, J. Llibre and M. Ollé
  Central configurations of the planar coorbital satellite problem
  Celestial Mech. Dynam. Astronom. 89 (2004), 319-342.
- M. Corbera and J. Llibre
  Central configurations of nested regular polyhedra for the spatial 2n-body problem
  J. Geom. Phys. 58 (2008), 1241-1252.

Periodic orbits

E. Belbruno, J. Llibre and M. Ollé
On the families of periodic orbits which bifurcate from the circular Sitnikov motions
Celestial Mech. 60 (1994), 99-129.

J. Llibre, K. Meyer and J. Soler.
Bridges between the generalized Sitnikov family and the Lyapunov family of periodic orbits
JDE, 154 (1999), 1241-1252.

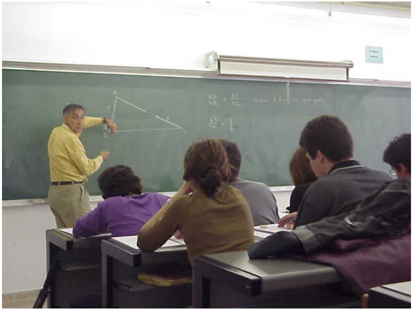
J. Llibre and R. Ortega
On the families of periodic orbits of the Sitnikov problem
SIAM J. Appl. Dyn. Syst. 7 (2008), 561-576.

J. Llibre and L. A. Roberto
On the periodic orbits and the integrability of the regularized Hill lunar problem
J. Math. Phys. 52 (2011), 082701.

Jaume's contributions on Discrete Dynamical Systems

(brief summary)

J. Llibre and A. Reventós
On the number of fixed points for a continuous map of a finite connected graph
Collect. Math., 32 (1981), 203-219.



Ll. Alsedà, J. Llibre and M. Misiurewicz
Periodic orbits of maps of Y
TAMS, 313 (1989), 475-538.

We denote

$$G(n) = \{n\} \cup \{k : k >_g n\} \quad \text{for } n \in \mathbf{N} \setminus \{2\},$$
$$R(n) = \{n\} \cup \{k : k >_r n\} \quad \text{for } n \in \mathbf{N} \setminus \{2, 4\},$$

and additionally

$$G(3.2^\infty) = R(3.2^\infty) = \{1\} \cup \{3.i : i \in S(2^\infty)\}.$$

We also denote $\mathbf{N}_g = (\mathbf{N} \setminus \{2\}) \cup \{3.2^\infty\}$ and $\mathbf{N}_r = (\mathbf{N} \setminus \{2, 4\}) \cup \{3.2^\infty\}$.

Main Theorem. (a) *If* $f \in \mathbf{Y}$, *then* $\mathrm{Per}(f) = S(n_s) \cup G(n_g) \cup R(n_r)$ *for some* $n_s \in \mathbf{N}_s$, $n_g \in \mathbf{N}_g$, *and* $n_r \in \mathbf{N}_r$.

(b) *If* $n_s \in \mathbf{N}_s$, $n_g \in \mathbf{N}_g$, *and* $n_r \in \mathbf{N}_r$ *then there exists* $f \in \mathbf{Y}$ *for which* $\mathrm{Per}(f) = S(n_s) \cup G(n_g) \cup R(n_r)$.

J. Llibre and M. Misiurewicz
Horseshoes, entropy and periods for graph maps *(or Excess of gods implies chaos)*
Topology, 32 (1993), 649-664.

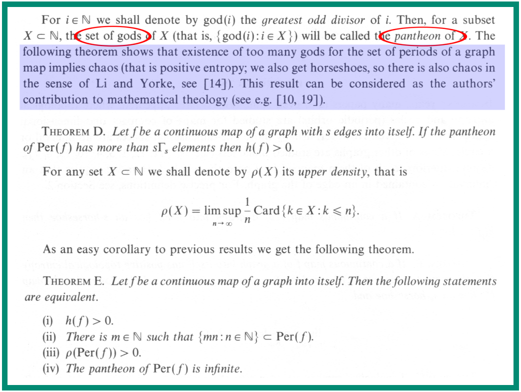
THEOREM A. *If a continuous map f of a graph into itself has an s-horseshoe then* $h(f) \geqslant \log s$.

THEOREM B. *If a continuous map f of a graph into itself has positive topological entropy then there exist sequences $(k_n)_{n=1}^{\infty}$ and $(s_n)_{n=1}^{\infty}$ of positive integers such that for each n the map f^{k_n} has an s_n-horseshoe and*

$$\limsup_{n\to\infty} \frac{1}{k_n} \log s_n = h(f).$$

THEOREM C. *Topological entropy, as a function of a continuous map of a graph into itself, is lower semi-continuous.*

J. Llibre and M. Misiurewicz
Horseshoes, entropy and periods for graph maps *(or Excess of gods implies chaos)*
Topology, 32 (1993), 649-664.


For $i \in \mathbb{N}$ we shall denote by $\text{god}(i)$ the *greatest odd divisor* of i. Then, for a subset $X \subset \mathbb{N}$, the set of gods of X (that is, $\{\text{god}(i) : i \in X\}$) will be called the *pantheon* of X. The following theorem shows that existence of too many gods for the set of periods of a graph map implies chaos (that is positive entropy; we also get horseshoes, so there is also chaos in the sense of Li and Yorke, see [14]). This result can be considered as the authors' contribution to mathematical theology (see e.g. [10, 19]).

THEOREM D. *Let f be a continuous map of a graph with s edges into itself. If the pantheon of* $\text{Per}(f)$ *has more than* $s\Gamma_s$ *elements then* $h(f) > 0$.

For any set $X \subset \mathbb{N}$ we shall denote by $\rho(X)$ its *upper density*, that is

$$\rho(X) = \limsup_{n \to \infty} \frac{1}{n} \text{Card}\{k \in X : k \leqslant n\}.$$

As an easy corollary to previous results we get the following theorem.

THEOREM E. *Let f be a continuous map of a graph into itself. Then the following statements are equivalent.*

(i) $h(f) > 0$.
(ii) *There is* $m \in \mathbb{N}$ *such that* $\{mn : n \in \mathbb{N}\} \subset \text{Per}(f)$.
(iii) $\rho(\text{Per}(f)) > 0$.
(iv) *The pantheon of* $\text{Per}(f)$ *is infinite.*

- Sets of periods in surfaces:

J. Franks and J. Llibre
Periods of surface homeomorphisms
Continuum Theory and DS (Arcata, CA), (1987) 63-77.

J. Llibre
Periodic point free continuous self-maps on graphs and surfaces
Topology Appl. 159 (2012), 2228-2231.

- Types of orbits (braid types):

J. Llibre and R. S. MacKay
Pseudo-Anosov homeomorphisms on a sphere with four punctures have all periods
Math. Proc. Cambrige Philos. Soc., 112 (1992), 539-549.

A continuous research activity since 1992.

- Homotopy (minimal periods):

J. Llibre and V. F. Sirvent
Minimal sets of periods for Morse-Smale diffeomorphisms on orientable compact surfaces
Houston J. Math., 35 (2009), 835-855.

J. Llibre
Periodic point free continuous self-maps on graphs and surfaces
Topology Appl., 159 (2012), 2228-2231.

A continuous research activity since 1992.

- A special case (minimal periods): The torus

B. Jiang and J. Llibre
Minimal sets of periods for torus maps
Discrete Contin. Dyn. Syst., 4 (1998), 301-320.

Ll. Alsedà, S. Baldwin, J. Llibre, R. Swanson and W. Szlenk
Minimal sets of periods for torus maps via Nielsen numbers
Pacific J. Math., 169 (1995), 1-32.

Jaume's contributions on Qualitative Theory of Differential Equations

(brief summary)

- B. Coll A. Gasull, J. Llibre
  Some theorems on the existence, uniqueness, and nonexistence of limit cycles for quadratic systems
  JDE, 67 (1987), 37-399.
- A. Gasull and J. Llibre
  Limit cycles for a class of Abel equations
  SIAM J. Math. Anal. 21 (1990), 1235-1244.
- H. Giacomini, J. Llibre and M. Viano
  On the nonexistence, existence and uniqueness of limit cycles
  Nonlinearity, 9 (1996), 501-516.

- F. Chen, C. Li, J. Llibre and Z. Zhang
  A unified proof on the weak Hilbert's 16th problem for n=2
  JDE, 221 (2006), 309-342.

- Y. Ilyashenko and J. Llibre
  A restricted version of the Hilbert's 16th problem for quadratic vector fields
  Mosc. Math. J. 10 (2010), 317-335.

- C. Li and J. Llibre
  Uniqueness of limit cycle for Liénard differential equations of degree four
  JDE (2012).

A. Gasull, S. Li-Ren and J. Llibre.
Chordal quadratic systems
Rocky Mountain J. Math. 6 (1986), 751-782.

- A. Gasull, S. Li-Ren and J. Llibre.
  Chordal quadratic systems
  Rocky Mountain J. Math. 6 (1986), 751-782.
- A. Cima and J. Llibre
  Bounded polynomial vector fields
  Trans. Amer. Math. Soc. 318 (1990), 557-579.
- J. C. Artés, B. Grünbaum and J. Llibre
  On the number of invariant straight lines for polynomial differential systems
  Pacific J. Math. 184 (1998), 207-230.
- J. Llibre, J. Sotomayor.
  Phase portraits of planar control systems
  Nonlinear Anal. 27 (1996), 1177-1197.

- A. Gasull, J. Llibre and J. Sotomayor
  Global asymptotic stability of differential equations in the plane
  JDE, 91 (1991), 327-335.
- J. C. Artés, R. E. Kooij and J. Llibre
  Structurally stable quadratic vector fields
  Mem. AMS 134 (1998).
- J. Chavarriga, H. Giacomini, J. Giné, J. Llibre
  Local analytic integrability for nilpotent centers
  Ergodic Theory Dynam. Systems 23 (2003), 417-428.

J. Llibre
Integrability of polynomial differential systems
Handbook of Differential Equations (2004) 437-532.

J. Llibre and X. Zhang
Invariant algebraic surfaces of the Lorenz system
J. Math. Phys., 43 (2002), 1622-1645.

J. Llibre and C. Valls
Integrability of the Bianchi IX system
J. Math. Phys, 46 (2005), 1-13.

C. J. Christopher, J. Llibre, C. Pantazi and S. Walcher
Darboux integrating factors: inverse problems
JDE, 250 (2011), 1-25.

Jaume's llibres (ups! not the family name: books)

- Ll. Alsedà, J. Llibre and M. Misiurewicz
  Combinatorial dynamics and entropy in dimension one
  World Scientific Publishing Co. Inc., River Edge NJ, 2000.

  $$\text{Warszawa} \quad \Longleftrightarrow \quad \text{Barcelona}$$

- J. C. Artés, F. Dumortier and J. Llibre
  Qualitative theory of planar differential systems
  Springer-Verlag, Berlin, 2006.

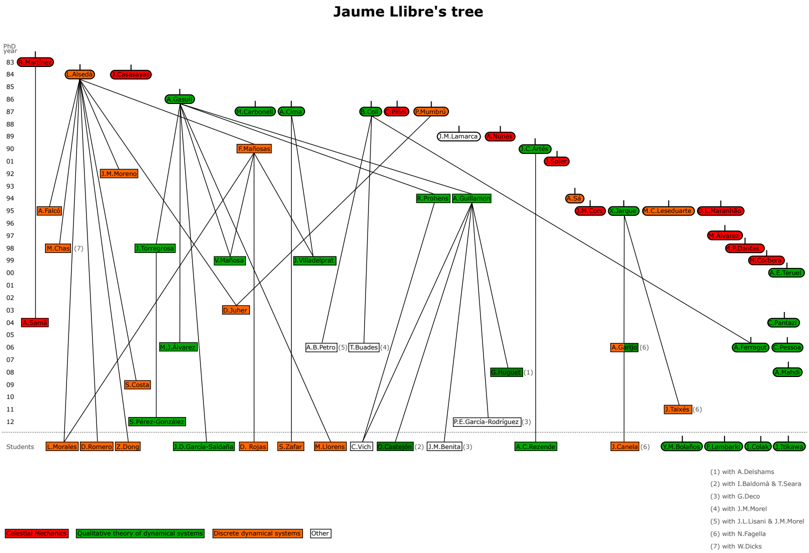
  $$\text{Hasselt} \quad \Longleftrightarrow \quad \text{Barcelona}$$

- J. Llibre and A. E. Teruel
  Qualitative theory of planar continuous piecewise linear systems
  to appear Springer-Verlag.

- J. C. Artés, N. Vulpe D. Schlomiuk and J. Llibre
  TBA
  TBA.

Countries in Jaume's work

Jaume's tree (students and descendents)

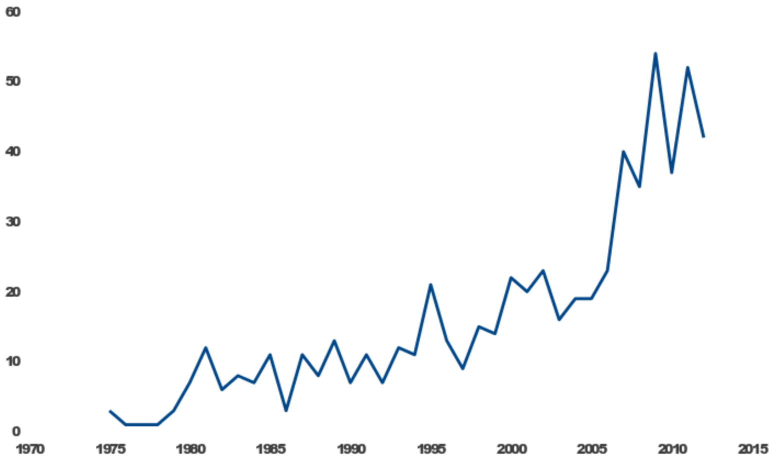


Coauthors evolution

Some numerics....(We use Taylor series of order big enough)

- PhD students: 26 (10 CM, 4DDS, 11QT) + 4 (present)
- Total PhD descendents: 46 (11 CM, 10DDS, 19 QT + 6 OTHERS)
- Coauthors: $\approx$ 215
- Articles (of the tree): $\approx$ 1000.
- Jaume's articles: $\approx$ 600.

Some numerics....(We use Taylor series of order big enough)



Some numerics....(We use Taylor series of order big enough)

- People who cite Jaume (in MathSciNet): ≈ 800.
- Number of cites of his work (in MathSciNet): ≈ 2200.
- Cites of Jaume's book on DDS (in MathSciNet): ≈ 160.
- Cites of Jaume's book on CDS (in MathSciNet): ≈ 60.
- Cites of Jaume's most cited paper (in MathSciNet): ≈ 60.

To be continued...

Family Theorem