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Transfers from LEOs to GEOs visiting libration points in the Sun-Earth RTBP

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Abstract

The main objective of the work is to find low cost transfer trajectories from a LEO orbit to a GEO orbit, which will have passages around a neighborhood of a libration point. Instead of using the invariant manifolds of the central manifold of the libration points, we proceed as follows. We consider the Sun-(Earth+Moon) Restricted Three Body Problem. Firstly, we obtain a catalogue of initial conditions that leave a neighborhood of the Earth at a certain altitude and such that their orbits reach a neighborhood of a libration point L_1 or L_2. That orbits are admissible to be *captured* and, with a small manoeuver, inserted in a stationary orbit around the equilibrium point, if needed. The orbits at a LEO altitude (approximately at 7000 km from the center of the Earth), are integrated forwards in time, while the orbits at a GEO altitude (approximately at 42000 km) are integrated backwards. Secondly, a matching-refinement procedure is used in order to find, among both catalogue of orbits, those that agree in positions, so that a difference Δv in velocities is obtained. The last objective is to identify the trajectories with the small manoeuvre such that can be used as a transfer from a LEO to a GEO orbit.

1. Introduction

Goal: find low-cost trajectories to transfer from low altitudes to high altitudes visiting a neighborhood of the equilibrium point L_1 or L_2.

Motivations:

- Transfers from low Earth orbits (LEO) to geostationary Earth orbits (GEO):
  The least speed change (Δv) using a direct transfer between two coplanar circular trajectories can be obtained using a Hohmann transfer by means of an elliptic orbit which is tangent to both circular orbits. Supposing all the orbits in the same plane, a manoeuvre at the perigee and the apogee must be done with a Δ_v equal to the difference between the velocity within the circular orbits (v_c) and the outgoing (v_o) and incoming (v_i) velocities.

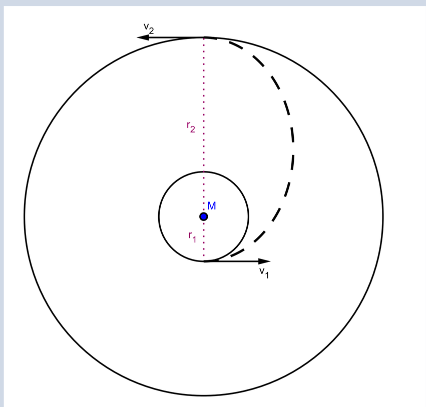


Figure : Hohmann transfer between two coplanar circular orbits

$$\Delta v_1 = v_{o_1} - v_{c_1} = \sqrt{2\left(\frac{\mu}{r_1} - \frac{mu}{r_1+r_2}\right)} - \sqrt{\frac{\mu}{r_1}}$$
$$\Delta v_2 = v_{c_2} - v_{i_2} = \sqrt{\frac{\mu}{r_2}} - \sqrt{2\left(\frac{\mu}{r_2} - \frac{mu}{r_1+r_2}\right)}$$

  Consider LEO orbits of about $600-650$ km, and a GEO orbit at 35786 km. Taking the two body approximation and circular orbits of about $r_1 = 7000$ km and $r_2 = 42000$ km, the Hohmann transfer will produce a total $\Delta v \simeq 3.768$ km/s, in approximately 5 hours and 18 minutes ([1]).
- Lunar transfers:
  GRAIL orbiters follow a low-energy transfer trajectory to the Moon that leaves the Earth towards the Sun passing near L_1 Sun-Earth Lagrange point before heading back towards the Earth-Moon system.

Previous results:
In ([3]), the author looks for low energy transfers from a specific LEO to a given GEO. Starting from suitable initial conditions on the LEO, integrate forwards until a neighbourhood of L_1 is reached in less than a maximum number of days. The same is repeated from the GEO integrating backwards. Then, the two trajectories are inserted in a Lyapunov orbit with respective manoeuvres in order join them and to have a trajectory that goes from a LEO orbit to a GEO orbit saving energy. The Δv required are around 1.2 km/s and 1.9 km/s.

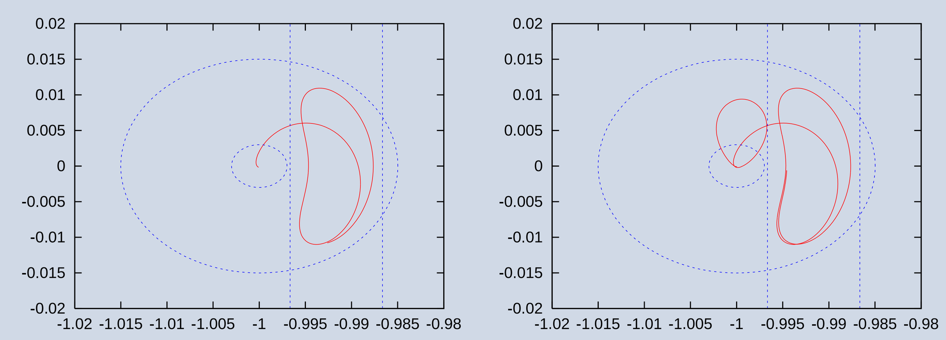
Context:
We consider the Restricted Three Body Problem (RTBP) Sun+Earth+spacecraft, and the equilibrium point L_1 located in between the two main bodies.

2. Transfers to L_1

It is known that the invariant manifolds $W^{u/s}$ associated to the objects in the central manifold of a collinear point are the main responsible of the dynamics around them. For the Sun-Earth RTBP, these manifolds have close passages to the Earth for certain values of the energy ([2]). Their role in the dynamics allow to find i.c. such that after some time the orbit reach and stays some time within a neighborhood of the equilibrium point.

Methodology:
Fix a value of the Jacobi constant C_J (energy) close and small than $C_J(L_1) \simeq 3.00089$.

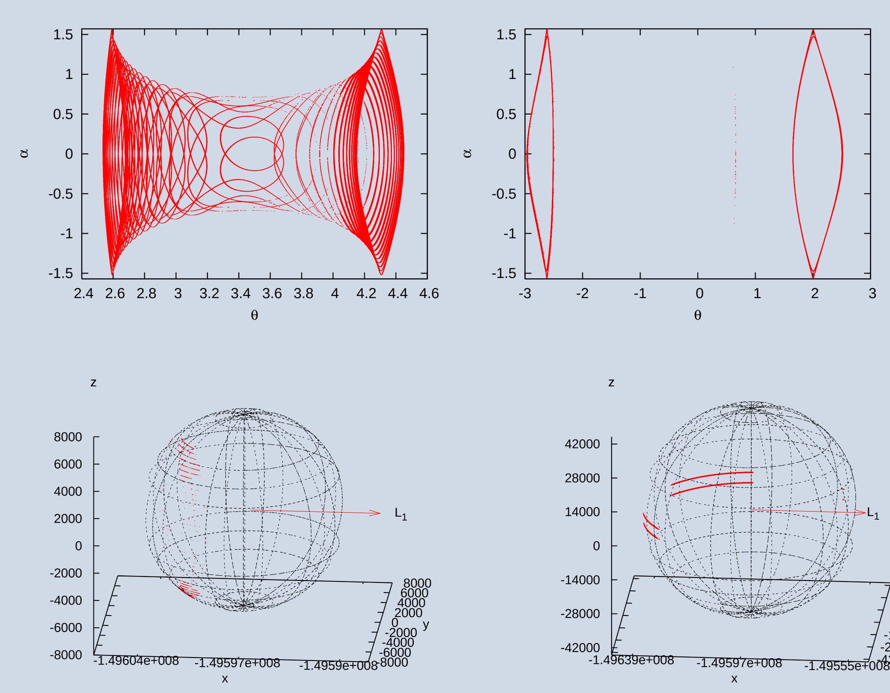
- Consider a set of initial conditions on a sphere of radius r_1 around the centre of Earth
$$\begin{aligned} x &= \mu - 1 + r_1\cos\varphi\cos\theta & x' &= v\cos\alpha\cos\beta \\ y &= r_1\cos\varphi\sin\theta & y' &= v\cos\alpha\sin\beta \\ z &= r_1\sin\varphi & z' &= v\sin\alpha \end{aligned}$$
  leaving the sphere tangentially $\vec{r}\cdot\vec{v} = 0$:
$$\cos\varphi\cos\alpha\cos(\theta-\beta) + \sin\varphi\sin\alpha = 0.$$
- Vary uniformly longitude and latitude
$$\varphi \in (-\pi/2, \pi/2), \quad \theta \in (-\pi, \pi).$$
- At each position, v is obtained from the Jacobi constant. Vary uniformly $\alpha \in [-\bar{\alpha}, \bar{\alpha}]$, where $\tan\bar{\alpha} = \frac{1}{|\tan\varphi|}$. Then β can take two values
$$\beta = \theta \pm \arccos(-\tan\varphi\tan\alpha).$$
- Integrate the i.c. **forwards** and keep the orbits that reach and stay inside a region $\mathcal{C}_{L_1}$ around the equilibrium point for a certain time.



- Repeats the same steps above starting at a distance $r_2 > r_1$ and integrating **backwards**.

The same procedure can be repeated towards L_2.

Results: We consider r_1 and r_2 values (adimensional units) corresponding to 7000 km (left) and 42000 km (right).



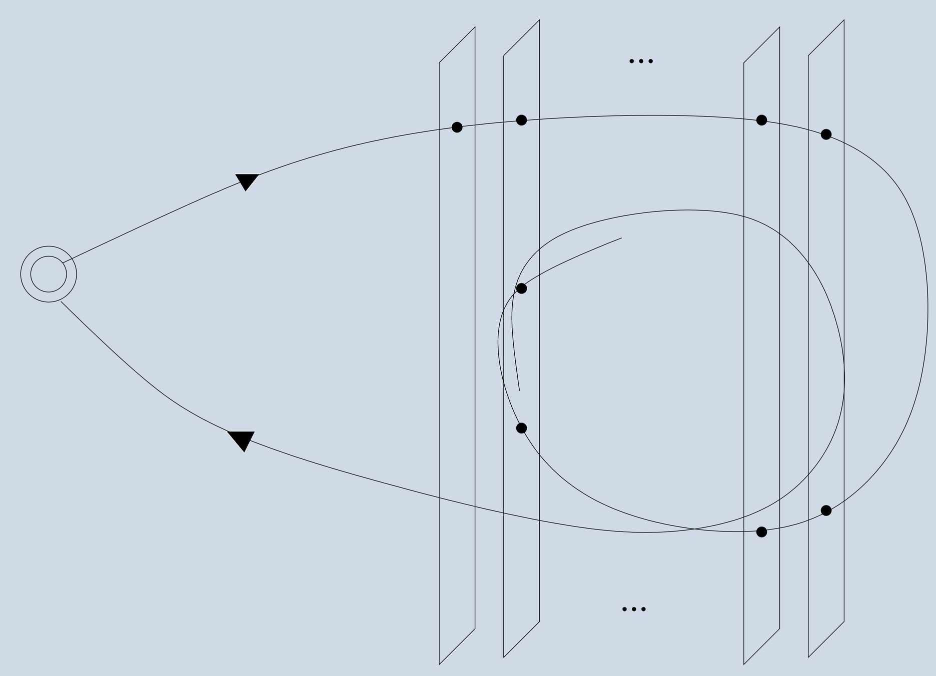
3. Matching

The main objective of the matching is to find initial conditions of low and high altitude orbits that match in position somewhere within $\mathcal{C}_{L_1}$ with the small possible Δv.

In general, for the same value of C_J, the range of i.c. (in latitude and longitude) that reach the box around L_1 is bigger for the set at 7000 km than for the set at 42000 km. For the i.c. leaving the sphere at r_1 any latitude is admissible, whereas for the i.c. at r_2 (arrival points) we are interested in latitudes around $\pm 23°$ (close to GEO).

Methodology:

- Consider i.c. of a LEO orbit at r_1 that reach L_1 integrating forwards.
- Find the intersections of the orbit with a sequence of sections $x = x_j$, $j = 1, \dots, n$ for a fixed values x_j within the region $\mathcal{C}_{L_1}$ defined in the previous section.
- Look for an orbit with i.c. at r_2 with the same energy as the LEO, such that integrated backwards have a close passage in position to any of the intersection points obtained in the step before.
- Refine (in variables ϕ, θ, α) the orbit to find a matching point (in position) of the two orbits.
- Use a continuation procedure to obtain a set of orbits that *match* in positions: vary the initial LEO i.c., and use the solution of the previous refinement to find a new pair of matching orbits.
- For each pair of orbits, a difference in velocities Δv is obtained.



Preliminary results:

- The advantage of going to the point L_1 is that the modulus of the velocity of the final point in the region $\mathcal{C}_{L_1}$ is less than 10^{-2}, so that
$$\Delta v = ||\vec{v}_i - \vec{v}_o|| \le 2\max(||v_i||, ||v_o||) \le 2\times 10^{-2}.$$
- From the initial explorations we obtain orbits that match in positions with a differ of 10^{-7} RTBP units (about 15 km) and a difference in velocities of 10^{-3} (30 m/s).
- The time spent from the i.c. starting at 7000 km and 42000 km is of order 2.5 and 1, respectively (ad. units), which represents a total time of 175–200 days.

Conclusions and work in progress

- The main objective is use the collinear points L_1 and L_2 to find *low-cost* trajectories from LEO to high altitude orbits visiting a neighborhood of an equilibrium point.
- Preliminary results show that two such transfers can match in positions up to 15 km, obtaining a small Δv in velocities, although there is a price to pay: the amount of time can be of order of months.
- The exploration of the whole data obtained in the first part (transfers to L_1) is still in process in order to find better matching orbits (with smaller Δv and difference in positions) and look for the shortest in time.

References

[1] R. Bate, D. Mueller, and J. White. *Fundamentals of Astrodynamics*. Dover Publications, 1971.

[2] G Gómez, A Jorba, J.J. Masdemont, and C Simó. Study of the Transfer from the Earth to a Halo Orbit Around the Equilibrium Point L1. *Celestial Mechanics*, 56(4) 541-562, 1993.

[3] E. Herrera. Study of leo to geo transfers via the l_1 sun-earth or earth-moon libration points. Master's thesis, UPC, 2008.