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On generation of independent quadratic conserved quantities

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(in collaboration with Hans Lundmark)

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In a Hamiltonian system one can produce a conserved quantity from two conserved quantities by using the Poisson bracket. Jacobi considered this remark as the “deepest discovery by Poisson”, while other authors, as Bertrand, remarked that nobody ever discovered a new conserved quantity by using this process.

Hans Lundmark observed a more spectacular way of producing new conserved quantities from two given ones. With his advisor Stefan Rauch-Wojciechowski, they considered another class of equations, that they call the Newton systems, where, in a vector space of dimension $n$, a force depends on the position and defines the second derivative of the position with respect to time. Then two conserved quantities which are quadratic in the velocities produce $n - 2$ other ones. The theorem also works on a spherical space. In the Neumann problem on an $n$-dimensional sphere, starting with the energy and another quadratic conserved quantity, one produces in this way a (known) system of $n$ quadratic independent conserved quantities in involution.

Recently, we found with Lundmark a simple criterion for the functional independence of conserved quantities produced in such a way. We present the result quite simply, using the “projective dynamics” point of view, i.e. the properties of central projection in dynamics discovered by Appell in 1890.

Local Integrability and Linearizability of 3D Lotka-Volterra Systems

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We consider the problem of local integrability of three dimensional Lotka-Volterra systems at the origin. In two dimensions, the problem has been the subject of a number of investigations. In three dimensions, the possible mechanisms underlying integrability are more complex and the problems arising are computationally much harder. We report on recent work which gives necessary and sufficient conditions for integrability in the case of $(a: -b: c)$-resonance where $a + b + c \leq 4$. We also consider the applicability of the monodromy method to integrability problems for these systems.
Relaxation oscillations in slow-fast systems

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The talk deals with two-dimensional slow-fast systems. These systems depend on a small parameter $\epsilon$, and possibly also on other parameters, in a way that for $\epsilon = 0$ the equation has a continuum of singular points.

Such systems can be studied by means of Geometric Singular Perturbation Theory. This theory essentially relies on center manifold reduction. The first to introduce it was Fenichel. Traditional Fenichel theory can however only be used near normally hyperbolic situations. Around 1995 it became clear how the blow technique could extend the power of geometric singular perturbation theory to including contact points.

Center manifolds, normal forms and blow-up permit to treat singular perturbation problems by means of traditional methods from dynamical systems theory.

In the talk we will only shortly recall the essential ingredients from the theory. We will mainly present a number of recent results concerning relaxation oscillations for $\epsilon > 0$, $\epsilon$ small: their number, the bifurcations they undergo.

The results come from a number of papers of Robert Roussarie, Peter De Maesschalck and myself.

Algebraic moments: from Abel equations to Jacobian conjecture

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The classical Hilbert’s 16th problem on limit cycles of polynomial vector fields impelled interest in 1-dimensional non autonomous systems with Abel equations.

In the setting of Abel equations, perturbation theory yields the algebraic moment problem (posed by M. Briskin, Y. Yomdin and JPF). The generic case was first solved by C. Christopher and the full problem was latter solved by Pakovich and Muzychuk. Related problems appeared in other fields of mathematics.

A similar problem in representation theory was posed by O. Mathieu. The general form of Mathieu conjecture implies the Jacobian conjecture. An important special case has been solved by Duistermaat and van der Kallen. Recently W. Zhao proposed several extensions of Mathieu conjecture related with powers of differential operators and orthogonal polynomials. The talk will be based on possible extensions to any dimensions of the algebraic moment problem as developped in [1].

References

Symplectic surface diffeomorphisms

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Suppose $M$ is a compact oriented surface of genus 0. We establish a structure theorem for area preserving diffeomorphisms of $M$ with zero entropy and at least three periodic points. As an application we show that rotation number is defined and continuous at every point of a zero entropy area preserving diffeomorphism of the annulus.

Further applications give insight into the algebraic structure of $\text{Symp}_\mu^\omega(M)$, the group of analytic symplectic diffeomorphisms of $M$. We show that if $G$ is a subgroup of $\text{Symp}_\mu^\omega(M)$ which has an infinite normal solvable subgroup, then $G$ is virtually abelian. In particular the centralizer $\text{Cent}(f)$ of an infinite order $f \in \text{Symp}_\mu^\omega(M)$ is virtually abelian. Another immediate corollary is that if $G$ is a solvable subgroup of $\text{Symp}_\mu^\omega(M)$ then $G$ is virtually abelian.

Uniqueness of limit cycles for Liénard differential equations of degree four

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A. Lins, W. de Melo and C. C. Pugh [4] conjectured that the classical Liénard differential equation of degree $n$ has at most \( \left\lfloor \frac{n-1}{2} \right\rfloor \) limit cycles, and they proved that the conjecture is true for $n = 3$. F. Dumortier, D. Panazzolo and R. Roussarie [2] gave a counterexample to this conjecture for $n = 7$ and they mentioned that it can be extended to $n \geq 7$ odd. Recently, P. De Maesschalck and F. Dumortier [1] proved that the classical Liénard differential equation of degree $n \geq 6$ can have \( \left\lfloor \frac{n-1}{2} \right\rfloor + 2 \) limit cycles. Xianwu Zeng [5] found a sufficient condition to guarantee the uniqueness of limit cycles for a subclass of classical Liénard differential equations of degree four.

In the talk we introduce a recent result [3] that any classical Liénard differential equation of degree four has at most one limit cycle, and the limit cycle is hyperbolic if it exists. This gives a positive answer to the above conjecture for $n = 4$. 
The three million body problem

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In recent years, J. Llibre and others from the celestial mechanics community have been using the tools we have developed to examine serious astrodynamics concerns. In this talk, a different, important challenge for our community will be described: it involves the about 1100 satellites that need to be protected from collisions with around three million pieces of space debris. This difficult, complex concern was the topic of a September 2012 US National Research Council report; aspects of this report will be described.
The stability properties of Hill’s linear periodic ODE for large parameters

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The goal is to study the parameter plane in the large for Hill-like equations, that is, of the form \( \ddot{x} + (a + bp(t))x = 0 \), \( p \) being 1-periodic (or 2π-periodic) with zero average.

Asymptotic estimates of the density of the stability regions in the \((a,b)\)-plane for lines of the form \( a = \omega^2 \cos(\psi), b = \omega^2 \sin(\psi) \) when \( \omega \to \infty \) are provided.

This density changes in a discontinuous way at some critical values of \( \psi \) and the fine structure across these critical directions is investigated.

Furthermore an explanation is given for the web-like structure of the exponentially narrow stability channels, for large \( a, b \), together with asymptotic estimates of the lines forming that web.

The talk is partly based on ongoing work with H. Broer and M. Levi.

Abelian integrals of general rational 1-forms are defined over \( \mathbb{Q} \)

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If a planar vector field with a polynomial Hamiltonian \( H \in \mathbb{R}[x,y] \) is perturbed in a polynomial 1-parametric family

\[
\dot{x} = \frac{\partial H}{\partial y} + \varepsilon Q(x,y), \quad \dot{y} = -\frac{\partial H}{\partial x} - \varepsilon P(x,y),
\]

then the limit cycles which bifurcate from a nonsingular oval on a level curve \( \{H(x,y) = z\}, z \in \mathbb{R} \), correspond to the zeros of the Abelian integral

\[
0 = \oint_{H=z} P(x,y) \, dx + Q(x,y) \, dy, \quad P, Q \in \mathbb{R}[x,y]. \quad (1)
\]

Establishing an explicit upper bound for the number of isolated zeros (roots) of the integral (1) in terms of the degrees of the polynomials \( H, P, Q \) was called the infinitesimal Hilbert 16th problem. While many
low degree cases where well studied since late 1960-ies, when the problem was first formulated, the general double exponential bound was achieved only in 2010 [1].

However, the result of [1] fails to address the case where both the Hamiltonian and the perturbation form $\omega = Pdx + Qdy$ are merely rational, not necessarily polynomial. While the proof of [1] can be relatively easily modified to cover the case of a rational Hamiltonian $H$, the appearance of poles for the form $\omega$ is considerably more difficult to overcome. One of the immediate reasons is that, while integrals of polynomial 1-forms of degree $\leq d$ form a finite-dimensional linear space, the integrals of rational 1-forms do not.

Dynamically, the rational perturbations naturally appear in the study of integrable polynomial vector fields with non-isolated singularities. Some of the simplest cases with the quadratic first integral where considered by J. Llibre in many publications with various co-authors (see, e.g., [3, 4]). They discovered quite a few peculiar properties of the integral (1) with a rational form $\omega$ as an analytic function of $z$. For instance, this function generically has ramification points of finite order, whereas a generic polynomial integral (1) has only logarithmic ramification points for finite values of $z$.

However, it turns out that even in the general rational case the integral (1) can be expressed via a suitable family of $Q$-functions, the class of transcendental functions defined by Pfaffian integrable systems with quasiumipotent monodromy over $\mathbb{Q}$ introduced in [2]. The key idea is to show that certain integrals satisfy a Picard–Fuchs-type system of equations with rational coefficients. Unlike the polynomial case where a simple algorithm of deriving such a system exists, in the rational case we only prove the existence of such a system along with an explicit upper bound for the complexity of its coefficients. This is sufficient to prove the double exponential upper bound for the number of isolated roots of a general Abelian integral, settling thus completely the infinitesimal Hilbert 16th problem for perturbations of foliations with algebraic first integrals.

References


Behavior of the binary collision in a planar restricted (N+1)-body problem

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We consider the planar restricted (N+1)-body problem, where the primaries are moving in a central configuration. It is verified that when the energy approaches minus infinity, then the infinitesimal mass \( m_1 \) is close enough to a primary, we use Levi-Civita and McGehee coordinates to regularize the binary collision. A canonical transformation is constructed which leaves the equations in the form of a perturbed resonant pair of harmonic oscillators where the perturbation parameter is the reciprocal of the energy. Firstly, it is proved the existence of four transversal ejection-collision orbits. After that the construction of the annulus mapping is carried out, and the condition of the Moser invariant curve theorem are verified, and then we are able to infer the existence of long periodic solutions for the restricted (N+1)-body problem. Also it is proved the existence of quasi-periodic solutions close to the binary collision. The first result implies, via KAM theorem, the existence, for certain intervals of values of the Jacobi constant, of an uncountable number of invariant punctured tori in the corresponding energy surface.

This work grew out of an attempt to carry over the methods of the study of the restricted three body problem for high values of the Jacobian constant by Conley [1], Chenciner [2] and Chenciner-Llibre [3] applying their techniques to a more general restricted problem. Our goal in this paper is to give a generalization of the Conley thesis results. In addition, we show that the Hill terms (the terms of sixth order) are of the same nature but with different coefficients, which allow us to give the differences with respect to known results. Thus we point out conditions on the relative equilibrium of the N-body problem in order to overcame the difficulties.

References


Semiconjugacy to a map of a constant slope - new results

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In one-dimensional dynamical systems there is a well known theorem claiming that any continuous piecewise monotone interval map $f$ with the positive topological entropy $h(f)$ is semiconjugated to some piecewise affine map with the constant slope $e^{h(f)}$ [6], [5], [1]. It is already known that analogous results remain true also for rich classes of Markov countably piecewise monotone continuous interval maps [4], [2]. Using the Vere-Jones classification of ergodic properties of infinite nonnegative matrices we prove new results in this direction [3].

References

A second order analysis of the periodic solutions for nonlinear
periodic differential systems with a small parameter

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We deal with nonlinear T–periodic differential systems depending on a small parameter. The unperturbed
system has an invariant manifold of periodic solutions. We provide the expressions of the bifurcation
functions up to second order in the small parameter in order that their simple zeros are initial values of
the periodic solutions that persist after the perturbation. In the end two applications are done. The key
tool for proving the main result is the Lyapunov–Schmidt reduction method applied to the T–Poincaré–
Andronov mapping. The results presented here are extensions to more general cases of the results from
[1].

References

Global instability in the elliptic restricted three body problem
using two scattering maps

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The goal of the talk is to show the existence of global instability in the elliptic restricted three body problem. The main tool is to combine two different scattering maps associated to the normally parabolic infinity manifold to build trajectories whose angular momentum increases arbitrarily. The computation of such scattering maps will rely heavily on the seminal computations for the circular case initiated first in Jaume Llibre’s thesis and finished later on by Llibre and Simó [1], which were extended to the elliptic case by Martínez and Pinyol [2].

References


Rotopulsating orbits in the curved N-body problem

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We consider the gravitational motion of \( N \) bodies of positive masses in spaces of constant curvature \( \kappa \neq 0 \), which can be reduced to the sphere \( S^3 \) for \( \kappa > 0 \) and the hyperbolic sphere \( \mathbb{H}^3 \) for \( \kappa < 0 \), [1], [4], [5]. The rotopulsating orbits are the analogue of the homographic solutions of the Euclidean case, i.e. the configuration of the bodies rotates and/or dilates or contracts, [2], [3]. In this talk we present some of the properties of the rotopulsating orbits and find several classes of such solutions.

References

On the connectivity of the Julia set for meromorphic entire maps

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In this work we complete the proof of the following theorem: If \( f \) is a meromorphic map with a disconnected Julia set then \( f \) has at least one weakly repelling fixed point (that is either repelling or parabolic with derivative exactly one). A nice corollary of this result is that the Newton’s method applied to an entire map has connected Julia set.

The proof is partially based on the solution of an old question about the existence of absorbing domains. Let \( U \) be a hyperbolic domain in \( \mathbb{C} \) and let \( f : U \to U \) be a holomorphic map. An invariant domain \( W \subset U \) is absorbing in \( U \) for \( f \) if for every compact set \( K \subset U \) there exists \( n = n(K) > 0 \), such that \( f^n(K) \subset W \).
Synchronisation predictions via extended phase response curves

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The phase of an oscillator such as a spiking neuron is one of the main indicators of the effects of external stimuli on the (membrane potential) dynamics. Experimentally, the phase advancement is mostly computed through phase response curves (PRCs) obtained from recordings of the time variations in reaching the next peak of the membrane potential; successful methods have been used to predict it by means of theoretical PRCs evaluated on the attractor (limit cycle). However, stimulation in transient states may induce phase advancements that differ from the predictions given in the asymptotic state. By computing the isochrons (curves of constant phase) in a vicinity of the limit cycle, we are able to accurately generalize the PRCs to the transient states and, as well, to provide a methodology to compute the phase advancement under any type of stimulus (weak or strong, instantaneous or long-lasting). In this communication, we would like to emphasize the combination of different dynamical systems approaches to an applied problem: our first inspiration being the application of Lie symmetries for time control both in planar centers and around limit cycles (see [2] and [3]), we have ended up by studying phase advancement in oscillators (see [4]) using geometric theory of invariant manifolds (see [1]). We will finish by illustrating the implications of our results to synchrony prediction in systems under high-frequency periodic stimuli by means of the study of rotation numbers for 2D maps derived from the extended PRCs. [5])

References


Topological and algebraic reducibility for patterns on trees

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We extend the classical notion of block structure for periodic orbits of interval maps to the setting of tree maps and study the algebraic properties of the Markov matrix of a periodic tree pattern having a block structure. We also prove a formula which relates the topological entropy of a pattern having a block structure with that of the underlying periodic pattern obtained by collapsing each block to a point, and characterize the structure of the zero entropy patterns in terms of block structures. Finally, we prove that an n-periodic pattern has zero (positive) entropy if and only if all n-periodic patterns obtained by considering the k-th iterate of the map on the invariant set have zero (respectively, positive) entropy, for each k relatively prime to n.

Oscillatory motions in the restricted circular planar three body problem

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In 1980 J. Llibre and C. Simó [1] proved the existence of oscillatory motions for the restricted planar three body problem, that is of orbits which leave every bounded region but which return infinitely often to some fixed bounded region. To prove their existence, they related them to the symbolic dynamics associated with a transverse homoclinic point. In their work they had to assume that the ratio between the masses of the two primaries was exponentially small with respect to the angular momentum. In the present work, we generalize their work proving the existence of oscillatory motions for any value of the mass ratio.
On the Darboux theory of integrability of non-autonomous polynomial differential systems

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In this work we unfold some differential algebraic aspects of Darboux first integrals of polynomial vector fields \([1, 4]\). An interesting improvement is that our approach can be applied both to autonomous and non-autonomous vector fields. We give a sufficient and necessary condition for the existence of a Darboux first integral of a specific form for a polynomial vector field with some known algebraic invariant hypersurfaces. For the autonomous case, the classical result of Darboux is obtained as a corollary \([3]\). For the non-autonomous case our characterization improves a known criterium of Llibre and Pantazi, \([2]\).

References


Communications

Distributional chaos for linear operators

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We will present several results on distributional chaos for linear operators on Fréchet spaces. More precisely, we will give a computable condition that characterizes distributional chaos for linear operators. In particular, an operator $T : X \rightarrow X$ on a Banach space $X$ is distributionally chaotic if and only if there are vectors $x \in X$ whose orbit under $T$ behaves extremely irregular, in the sense that there are subsets $A, B \subseteq \mathbb{N}$ whose upper density equals 1 such that, $\lim_{n \to \infty, n \in A} \|T^n x\| = 0$ and $\lim_{n \to \infty, n \in B} \|T^n x\| = \infty$.

Newtonian few-body problem central configurations with gravitational charges of both signs

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The Newtonian n-Body Problem is modified assuming positive inertial masses but different sign for the masses in the interacting force, which is assumed with the possibility of two different signs for the gravitational mass, according to the prescription: two masses with same sign attract one to the other, two masses of different sign repel one to the other. As in electrostatics the signed mass is called charge. The two body problem behaves as the similar Coulomb problem of charged particles in the Bohr model of the atom, where radiation effects are avoided. For two bodies any solution is a central configuration with almost same behavior that the Newton two-body problem. The 3-Body problem was considered without any particular surprise, we have no planar solution, and several different collinear solutions. The four body case of charged central configurations has only the planar [1] and collinear solutions.

References

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Communications

Hunting three nested limit cycles with only two linear foci

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When the aggregation of two linear differential systems defines a discontinuous planar vector field, the problem of determining the maximum number of nested limit cycles surrounding only one equilibrium point, is more challenging than in the continuous case, where it is possible to establish the existence at most of one limit cycle, see [1]. In fact, by considering a specific family of discontinuous differential systems with two linear zones sharing the equilibrium position, strong numerical evidence about the existence of three nested limit cycles was obtained very recently in [3], contrarily to what it had been conjectured in [4]. The example in [3] has a real unstable focus and a virtual stable focus sharing their location. A rigorous, computer assisted proof of the existence of such limit cycles has been obtained in [5], but some explanation on their generation mechanism was lacking.

We will show, thanks to the canonical forms introduced in [2], how to analytically prove the existence of such three limit cycles in more general cases, by combining adequately the two linear foci. The hunting of these limit cycles can be done by using two different approaches, both in a bifurcation spirit. We can perturb a crossing-sliding limit cycle that coexists with a non-hyperbolic periodic orbit at infinity, but also the three limit cycles can bifurcate from a higher degeneration at infinity. We will mainly pay attention to the first mechanism.

References


Relative equilibria in the four-vortex problem
with two pairs of equal vorticities

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We consider the set of relative equilibria in the four-vortex problem where two pairs of vortices have equal strength, that is, $\Gamma_1 = \Gamma_2 = 1$ and $\Gamma_3 = \Gamma_4 = m$ where $m \in \mathbb{R} - \{0\}$ is a parameter. Our main result is that for $m > 0$, the convex configurations all contain a line of symmetry, forming a rhombus or an isosceles trapezoid. The rhombus family exists for all $m$ but the isosceles trapezoid case exists only for $m$ positive. In fact, there exist asymmetric convex configurations when $m < 0$. In contrast with the Newtonian 4-body problem, where the main symmetry result stated above is still unproven, the equations in the vortex case are somewhat easier to handle, allowing for a complete classification of all solutions.

Classification of Lattes maps on $P^2$

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Lattes maps belong to a special family of holomorphic maps on the complex projective space, the so-called critically finite maps. They have very unique dynamical properties. However, we are concerned with their classification in this talk. Several years ago, Milnor gave a classification of Lattes maps on $P^1$. We will present a classification of Lattes maps on $P^2$. 
Chaos in delay differential equations
with applications in population dynamics

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In this talk we develop a geometrical method to detect the presence of chaotic dynamics in infinite
dimensional spaces. Our method enables us to give explicit conditions in our models without using
small/large parameters or hiperbolicity conditions. An application to the classical Lotka-Volterra model
with delay is given.

The results are contained in [1].

References

appear in DCDS series A.

The period function’s higher order derivatives

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We prove a formula for the $n$-th derivative of the period function $T$ in a period annulus of a planar
differential system. For $n = 1$, we obtain Freire, Gasull and Guillamon formula for the period’s first
derivative [1]. We apply such a result to hamiltonian systems with separable variables and other systems.

We give some sufficient conditions for the period function of conservative second order O.D.E.’s to be convex.

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The global topological classification of the Lotka-Volterra quadratic differential systems

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The planar Lotka-Volterra systems intervene in many areas of applied mathematics, so naturally there were several attempts to give the topological classification of these systems, the last one published in 2008. These classifications are far from satisfying because apart from containing errors or being incomplete, they are done without the use of adequate global tools and so we end up with a maze of tables containing numerous cases expressed in terms of inequalities of the coefficients of the systems which fail to convey to us the global phenomena involved. In this work, jointly done with Nicolae Vulpe, we base our classification on the global concept of configuration of invariant lines of the systems. After a first work classifying the systems in terms of their associated configurations of invariant lines, in the present work we take each class determined by a specific configuration and classify it topologically. The final result is stated in terms of algebraic invariants. We give necessary and sufficient conditions in terms of invariant polynomials, which can be computed using computer algebra, for obtaining each one of the specific phase portraits of this class.

References

Simultaneous linearization of a class of pairs of involutions with normally hyperbolic composition

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In this talk a result on simultaneous linearization for a class of pairs of involutions whose composition is normally hyperbolic is presented. This extends the corresponding result when the composition of the involutions is a hyperbolic germ of a $2D$–diffeomorphism (Teixeira, 1982). Inside the class of pairs with normally hyperbolic composition, we obtain a characterization theorem for the composition to be hyperbolic. In addition, related to the class of interest, we present the classification of pairs of linear involutions via linear conjugacy. It is worth to say that the problem of simultaneous behavior of diffeomorphisms have leaded to several interesting results in different settings. Among such results, we mention the Bochner-Montgomery theorem (see Montgomery-Zippin, 1955) which is a well-known and useful result about linearization of a compact group of transformations around a fixed point. This theorem is preceded by a related result by Cartan (1955). We also mention the article of Voronin (1982) where the classification of pairs of $2D$-involutions is also considered.

References

Interval translation maps of three intervals

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Interval translation maps (ITMs) are the non-invertible generalizations of interval exchanges. We show that any ITM of three intervals can be reduced either to a rotation or to a double rotation. As a consequence, we prove the finiteness conjecture for the ITMs of three intervals. Namely, the subset of ITMs of finite type is open, dense, and has full Lebesgue measure. The set of ITMs of infinite type is a Cantor set of zero measure and of Hausdorff dimension less than full.

References

Communications

Dynamics of trace maps motivated by applications in spectral theory of quasicrystals

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Since the discovery of quasicrystals by Schehtman et. al. in the early eighties, quasiperiodic models in mathematical physics have formed an active area of research. In the pioneering works of M. Kohmoto et. al. [1] and M. Casdagli [2], a strong relationship between trace maps and spectral properties of quasiperiodic Schrödinger operators was discovered.

We discuss the dynamics of the so-called Fibonacci trace map, associated to a prototypical quasiperiodic model, and demonstrate how it can be applied in the study of spectral properties of a class of quasiperiodic operators. The Fibonacci trace map is an analytic map on the three-dimensional Euclidean space exhibiting nontrivial behavior (hyperbolicity on some invariant two-dimensional surfaces and partial hyperbolicity on a three-dimensional submanifold of $\mathbb{R}^3$ foliated by these invariant surfaces). This in turn has strong implications in spectral theory of the associated quasiperiodic quantum Hamiltonians (discrete Schrödinger and Jacobi operators on $\ell^2(\mathbb{C})$), such as fractal structure of the operator spectrum, estimates on fractal dimensions, regularity of fractal dimensions and the like [3, 4, 5, 6, 7, 8].

References


The 16th Hilbert problem: a simple version on algebraic limit cycles

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For real planar polynomial differential systems there appeared a simple version of the 16th Hilbert problem on algebraic limit cycles: *Is there an upper bound on the number of algebraic limit cycles of all real planar polynomial vector fields of degree $n$?* In [1] Llibre, Ramírez and Sadovskaia solved the problem in the case of invariant algebraic curves generic for the vector fields. The same authors [2] also provided an upper bound on the number of algebraic limit cycles for polynomial vector fields having only nonsingular invariant algebraic curves.

In this talk we report our results [3], which solved the problem for planar polynomial vector fields either having only nodal invariant algebraic curves, or having only non–dicritical invariant algebraic curves.

References


Chaotic behavior of the solution $C_0$-semigroup of the von Foerster-Lasota equation in different phase spaces

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The usually so-called von Foerster-Lasota equation, or McKendrick equation, is one of the first age-dependent models in dynamics of population. We will see that the solution $C_0$-semigroup of this equation is chaotic on $L^p[0,1]$ and in the subspace of the Sobolev spaces $W^{1,p}(0,1)$ of functions vanishing at the origin, where $p \in [1, +\infty)$. The approach is different from that previously used by [2], since it is based on an accurate estimate of the behavior of the eigenvectors of the generator of the semigroup. The previous results have been generalized to the study of first order linear parabolic equation with a more general drift (for more details we refer the reader to [1]).

References


Limit cycles of a generalized Liénard differential equation via averaging theory

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We apply the averaging theory of first and second order to the generalized Liénard differential equations. Our main result shows that for any $n, m \geq 1$ there are differential equations of the form $\ddot{x} + cf(x, \dot{x})\dot{x} + c^2g(x, \dot{x})\dot{x} + x = 0$, with $f$ and $g$ polynomials of degree $n$ and $m$ respectively, having at most $\lfloor \frac{n}{2} \rfloor$ and $\max\{\lfloor \frac{n}{2} \rfloor, \lfloor \frac{m}{2} \rfloor\}$ limit cycles using the averaging theory of first and second order respectively.
Transfers from LEOs to GEOS visiting libration points in the Sun-Earth RTBP

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The main objective of the work is to find low cost transfer trajectories from a LEO orbit to a GEO orbit, which will have passages around a neighborhood of a libration point. Instead of using the invariant manifolds of the central manifold of the libration points, we proceed as follows. We consider the Sun-(Earth+Moon) Restricted Three Body Problem. Firstly, we obtain a catalogue of initial conditions that leave a neighborhood of the Earth at a certain altitude and such that their orbits reach a neighborhood of a libration point $L_1$ or $L_2$. That orbits are admissible to be captured and, with a small manoeuver, inserted in a stationary orbit around the equilibrium point, if needed. The orbits at a LEO altitude (approximately at 7000 km from the center of the Earth), are integrated forwards in time, while the orbits at a GEO altitude (approximately at 42000 km) are integrated backwards. Secondly, a matching-refinement procedure is used in order to find, among both catalogue of orbits, those that agree in positions, so that a difference $\Delta v$ in velocities is obtained. The last objective is to identify the trajectories with the small manoeuvre such that can be used as a transfer from a LEO to a GEO orbit.
A class of cubic Rauzy Fractals

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The Rauzy fractal is a compact subset of the space \( \mathbb{R}^{d-1} \), \( d \geq 2 \). It has a fractal boundary and it induces two kind of tilings of \( \mathbb{R}^{d-1} \), one of them is periodic and the other is auto-similar. Rauzy fractals are connected to many areas as substitution dynamical system, number theory among others.(see[1, 2, 3]).

In this work we study arithmetical and topological properties of two classes of Rauzy fractals (\( R \) and \( G \)) given by the polynomial \( x^3 - ax^2 + x - 1 \) where \( a \geq 2 \) is an integer. We give explicitly an automaton that generates the boundary of \( R \) and \( G \). With this we prove that \( R \) has 8 neighbours while \( G \) has always 6. Moreover in the case \( a = 2 \) we can give further information on the boundary of these sets.

References


On the maximum number of limit cycles of a class of generalized Liénard differential systems

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Applying the averaging theory of first, second and third order to one class generalized polynomial Liénard differential equation, we improve the known lower bounds for the maximum number of limit cycles that this class can exhibit.

More precisely, given the equation
\[ \begin{align*}
\dot{x} &= y + \sum_{k \geq 1} \varepsilon^k h_k^l(x), \\
\dot{y} &= -x - \sum_{k \geq 1} \varepsilon^k (f_k^m(x)y + g_k^m(x)),
\end{align*} \tag{1} \]

where for every \( k \) the polynomials \( h_k^l(x) \), \( g_k^m(x) \) and \( f_k^n(x) \) have degree \( l \), \( m \) and \( n \) respectively, and \( \varepsilon \) is a small parameter. We show that the following result.

**Theorem.** If for every \( k = 1, 2 \) the polynomials \( h_k^l(x) \), \( g_k^m(x) \) and \( f_k^n(x) \) have degree \( l \), \( m \) and \( n \) respectively, with \( l, m, n \geq 1 \), then for \( |\varepsilon| \) sufficiently small, the maximum number of medium limit cycles of the polynomial Liénard differential systems (1) bifurcating from the periodic orbits of the linear center \( \dot{x} = y, \dot{y} = -x \), using the averaging theory

(a) of first order is \( \tilde{H}_1(l, m, n) = \left\lfloor \frac{\max\{O(l), O(n + 1)\} - 1}{2} \right\rfloor = \max \left\{ \left\lfloor \frac{l - 1}{2} \right\rfloor, \left\lfloor \frac{n}{2} \right\rfloor \right\} \),

(b) of second order is \( \tilde{H}_2(l, m, n) = \left\lfloor \frac{\max\{E(l) + E(m), O(n) + E(m) + 1, O(l), O(n + 1)\} - 1}{2} \right\rfloor \),

(c) of third order is \( \tilde{H}_3(l, m, n) \geq \left\lfloor \frac{\max\{O(m + n), E(l + m) - 1\} - 1}{2} \right\rfloor \), and

(d) the three upper bounds for \( \tilde{H}(l, m, n) \) given in statements (a), (b) and (c) for some values of \( l, m \) and \( n \) are reached. So they cannot be improved.

Where \( E(k) \) is the largest even integer \( \leq k \), and \( O(k) \) is the largest odd integer \( \leq k \).
On the trigonometric moment problem

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The trigonometric moment problem arises from the study of one-parameter families of centers in polynomial vector fields. It asks for the classification of the trigonometric polynomials $Q$ which are orthogonal to all powers of a trigonometric polynomial $P$.

We show that this problem has a simple and natural solution under certain conditions on the monodromy group of the Laurent polynomial associated to $P$. In the case of real trigonometric polynomials, which is the primary motivation of the problem, our conditions are shown to hold for all trigonometric polynomials of degree 15 or less. In the complex case, we show that there are a small number of exceptional monodromy groups up to degree 30 where the conditions fail to hold and show how counter-examples can be constructed in several of these cases.

References


No periodic orbits for the Einstein-Yang-Mills equations

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The static, spherically symmetric Einstein-Yang-Mills equations with a cosmological constant $a \in \mathbb{R}$ are

$$\begin{align*}
\dot{r} &= rN, \\
\dot{W} &= rU, \\
\dot{N} &= (k - N)N - 2U^2, \\
\dot{k} &= s(1 - 2ar^2) + 2U^2 - k^2, \\
\dot{U} &= sWT + (N - k)U, \\
\dot{T} &= 2UW - NT,
\end{align*}$$

where $(r, W, N, k, U, T) \in \mathbb{R}^6$, $s \in \{-1, 1\}$ refers to regions where $t$ is a time-like respectively space-like, and the dot denotes a derivative with respect to $t$. See for instance [1] and the references quoted therein for additional details on these equations.

The physicists are mainly interested in the solutions of the differential system (1) with $r > 0$, see the middle of the page 573 of [1].

In this work we proved that system (1) has no periodic solutions when $r > 0$.

References

Extending geometric singular perturbation theory for ordinary differential equations with three time–scales

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Systems in nature, which are modeled by ordinary differential equations, often involve two or more different time scales. For instance, in biological literature we can find many examples of models which present such features. When systems present a clear separation in time scales, methods of approximations of slow–fast systems can be applied. Around 1980, geometric singular perturbation theory was introduced. The foundation of this theory was laid by Fenichel [1] and it essentially uses geometric methods from dynamical systems theory for studying the properties of solutions of the system. Note that for the singular perturbation problems studied by Fenichel [1] only two different time–scales can be derived: a slow and a fast ones.

In this poster we consider systems of ordinary differential equations involving three different time–scales. These systems are in general written in the form

\[ \varepsilon \dot{x} = f(x, \varepsilon, \delta), \quad \dot{y} = g(x, \varepsilon, \delta), \quad \dot{z} = \delta h(x, \varepsilon, \delta), \]

(1)

where \( x = (x, y, z) \in \mathbb{R}^3 \), \( \varepsilon \) and \( \delta \) are two independent small parameter \((0 < \varepsilon, \delta \ll 1)\), and \( f, g, h \) are \( C^r \) functions, where \( r \) is big enough for our purposes. Now, three different time–scales can be derived of the system (1): a slow time–scale \( t \), an intermediate time–scale \( \tau_1 := \frac{t}{\delta} \) and a fast time–scale \( \tau_2 := \frac{\varepsilon}{\delta} \).

We intend to develop a geometric theory, similar the one given by Fenichel [1], for systems of the form (1). We present in this poster some results in this direction.

References

Bifurcations of non-smooth vector fields on $\mathbb{R}^3$.
The Cusp-Fold singularity

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This article presents results concerning a class of generic one-parameter families of 3D piecewise-smooth vector fields. The dynamics of the so called fold-cusp singularity is studied and its bifurcation diagram is exhibited. Results about the asymptotical stability of systems presenting such singularities are also discussed.

Global dynamics of the May–Leonard system

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We talk about the integrability and the global dynamics of the May–Leonard model in $\mathbb{R}^3$, which describe the competition between three species and depending on two positive parameters $a$ and $b$. Specially we analyze the cases $a + b = 2$ and $a = b$ in the compactification of the positive octant. Roughly speaking, if $a + b = 2$ and $a \neq 1$ there are invariant topological half-cones by the flow of the system. These half-cones have vertex at the origin of coordinates and surround the bisectrix $x = y = z$, and foliate the positive octant. The orbits of each half-cone are attracted to a unique periodic orbit of the half-cone, which lives on the plane $x + y + z = 1$.

When $b = a \neq 1$ then we consider two cases. First, if $0 < a < 1$ then the unique positive equilibrium point attracts all the orbits of the interior of the positive octant. Second, if $a > 1$ then there are three equilibrium points in the boundary of positive octant, which attract almost all the orbits of the interior of the octant, we describe completely their basins of attractions.
Global phase portraits of cubic systems having a center simultaneously at the origin and at infinity

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The center problem is one of the celebrated problems in the qualitative theory of planar differential equations which is closely related to bifurcation problems of limit cycles. Only quadratic centers are completely known. In this poster we present a systematic classification of a 6-parameter family of cubic differential systems

\[
\begin{align*}
\dot{x} &= -y + ax^2 + bxy + cy^2 - y(x^2 + y^2), \\
\dot{y} &= x + ex^2 + fxy + gy^2 + x(x^2 + y^2),
\end{align*}
\]

having simultaneously a center at the origin and at infinity. First of all such family can be classified in a Hamiltonian subfamily \(\{a = b + 2g = f = 0\}\) and a reversible one \(\{a = c = f = 0\}\) Next for both classes global phase portraits are classified topologically. Finally it is illustrated how the techniques developed in this study can be used to construct polynomial vector fields with described phase portraits.
Nilpotent systems with an inverse integrating factor.
Center problem and integrability

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We characterize nilpotent systems whose lowest degree quasi-homogeneous term is \((y, \sigma x^n)^T\), \(\sigma = \pm 1\), which have an algebraic inverse integrating factor over \(\mathbb{C}((x, y))\). In such cases, we show that the systems admit an inverse integrating factor of the form \((h + \cdots)^q\) with \(h = 2\sigma x^{n+1} - (n + 1)y^2\) and \(q\) a rational number.

We prove that, for \(n\) even, the systems with formal inverse integrating factor are formally orbital equivalent to \((\dot{x}, \dot{y})^T = (y, x^n)^T\). In the case \(n\) odd, we give a formal normal form that characterizes them. As a consequence, we give the link among the existence of formal inverse integrating factor, center problem and integrability of the considered systems.

References

Study of the equilibrium points of the restricted three-body problem with an oblate primary

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We consider the restricted three–body problem with the more massive primary as an oblate spheroid. The primaries are moving in a Keplerian circular orbit about their center of mass, the equatorial plane of the oblate primary coinciding with the plane of motion of the binaries.

We study the dynamics of a third particle of infinitesimal mass in space under the gravitational attraction of the binary, looking at the existence and stability of the equilibrium points.

The planar problem has been studied by several authors. They have confirmed the existence of five equilibrium points, three of them being collinear and two in a triangular configuration, as in the problem without oblateness. The collinear ones are unstable in the Liapunov sense; in the interval of linear stability of the triangular ones it has been shown stability except for three values of the mass parameter and the critical mass [5], [7], [8]. These results follow from Arnold Theorem [2].

In the spatial case, a three-degree of freedom system, Arnold Theorem does not apply but we can still try to establish stability results in a weaker formulation such as formal stability and stability of finite type. To this end we use normal forms techniques and the theory developed in [4]. We expand the potential in power series up to fourth order in the oblateness parameter, the eccentricity of the spheroid.

This spatial problem has been studied with an approximation of the potential to second order in the oblateness parameter [5], [6]. Recently, Markellos and Douskos, found two new equilibrium points outside the equatorial plane, nearly above and below the oblate primary [3]. We hope to say something about this point.

References

In this poster we present the global dynamics of the first order planar polynomial differential system of degree 2, called Lev Ginzburg differential system,

\[ \begin{align*}
    x' &= \frac{dx}{dt} = y, \\
    y' &= \frac{dy}{dt} = (1 - \beta_1 y)(\gamma - \alpha x + \beta y),
\end{align*} \tag{1} \]

depending on four parameters: \( \alpha > 0, \beta_1 > 0, \gamma > 0 \) and \( \beta \in \mathbb{R} \).

Bellamy and Mickens [5] claimed that the Lev Ginzburg differential equation (1) can exhibit a limit cycle coming from a Hopf bifurcation. In [2] the authors shown that this differential equation has neither a Hopf bifurcation, nor limit cycles.

We note that the Lev Ginzburg system (1) has the invariant straight line \( y = 1/\beta_1 \). So, system (1) has at most one limit cycle, and if it exists then it is hyperbolic. By using Poincaré Compactification, we show that the only chance to equation (1) exhibit a limit cycle is that case where the limit cycle is a non-hyperbolic one.

**References**


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Some new results on Darboux integrable differential systems

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We deal with differential systems \( X \) of the form \( \dot{x} = P(x, y), \dot{y} = Q(x, y) \) of degree \( d \) having a Darboux first integral \( H \) and an inverse integrating factor \( V \). Our first main result compares a natural extension of the degree of \( V \) with \( d + 1 \).

**Theorem 1.** Let \( \Pi_1 = \prod_{i=1}^{p} \tilde{f}_i^{\lambda_i}, \Pi_2 = \tilde{g}/\prod_{i=1}^{p} \tilde{f}_i^{n_i} \). Let \( \delta(\prod g_i^{\alpha_i}) = \sum \alpha_i \deg g_i \).

(a) \( \delta(V) < d + 1 \) if and only if \( \delta(\Pi_2) > 0 \).
(b) \( \delta(V) = d + 1 \) if and only if either \( \delta(\Pi_2) < 0 \) and \( \Pi_1 \) is not constant, or \( \delta(\Pi_2) = 0 \).
(c) \( \delta(V) > d + 1 \) if and only if \( \delta(\Pi_2) < 0 \) and \( \Pi_1 \) is constant.

Moreover in all cases we have an expression of the characteristic polynomial in terms of some inverse integrating factor.

**Corollary.** The infinity is degenerate if and only if \( \delta(\Pi_1) = 0 \) and either \( \delta(\Pi_2) < 0 \) and \( \Pi_1 \) is not constant, or \( \delta(\Pi_2) = 0 \).

The remarkable values and remarkable curves of rational first integrals were first introduced by Poincaré and afterwards studied by several authors. It has been shown in the literature that the remarkable curves play an important role in the phase portrait as they are strongly related to the separatrices.

In our work we first define remarkable values and remarkable curves of Darboux first integrals and afterwards we state the following result.

**Theorem 2.** Suppose that system \( X \) has a Darboux first integral \( H \) which is not rational. Then \( V \) is a polynomial if and only if \( H \) has no critical remarkable values.
Stability of fixed points for periodic Hamiltonian systems

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We study the stability of equilibria for periodic Hamiltonian systems with one and a half degrees of freedom. We focus on systems coming from the second Newton’s Law and we show that equilibria are unstable solutions when the force depends on time periodically and it is increasing at the equilibria. We give conditions to determine when the equilibria have hyperbolic structure. We show some examples exhibiting the powerful of the above result.

Cyclicity of a simple focus via the vanishing multiplicity of inverse integrating factors

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We consider planar real analytic differential systems

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y),$$

defined in a neighborhood $U \subset \mathbb{R}^2$ of the origin. We assume that $(0, 0)$ is a simple focus, i.e., one monodromic singularity which after a (generalized) polar blow-up $(x, y) \mapsto (\theta, r)$ is transformed into a periodic orbit. In short, system (1) can be written as

$$\frac{dr}{d\theta} = F(r, \theta) = \sum_{i \geq t} F_i(\theta) r^i,$$

where $F(r, \theta)$ is analytic on the cylinder $C = \{(r, \theta) \in \mathbb{R} \times S^1 : |r| \text{small}\}$ with $S^1 = \mathbb{R}/\mathbb{Z}T$ where $T > 0$ is the minimum constant period associated to the polar change. Here we have $F(0, \theta) = 0$ for all $\theta \in S^1$. Consider inverse integrating factors $V(r, \theta)$ of (2), i.e., a function $V : C \to \mathbb{R}$ non-locally null and solution of

$$\frac{\partial V(r, \theta)}{\partial \theta} + \frac{\partial V(r, \theta)}{\partial r} F(r, \theta) = \frac{\partial F(r, \theta)}{\partial r} V(r, \theta).$$

It is well known (see [1] and [2]) that (2) has a unique (modulo multiplicative constants) inverse integrating factor $V(r, \theta)$ of class $C^\infty$ and non-flat at $r = 0$. Therefore $V$ admits the Taylor series $V(r, \theta) = \sum_{i \geq m} v_i(\theta) r^i$. 
In [3] it is proved that $\Pi(r_0) = r_0 + c_m r_0^m + O(r_0^{m+1})$ with $c_m \neq 0$ is the expression of the Poincaré map associated to the origin of (2).

We shall prove that $m \geq \ell \geq 1$. Moreover:

(a) $m = \ell$ if and only if $v_k(\theta)$ are constant for $k = m, \ldots, 2\ell - 1$.

(b) Assume $\ell \geq 2 + k$ with $k \geq 0$ a positive integer. If $\int_0^T F_\ell(\theta)\,d\theta = \int_0^T F_{\ell+1}(\theta)\,d\theta = \cdots = \int_0^T F_{\ell+k-1}(\theta)\,d\theta = 0$, but $\int_0^T F_{\ell+k}(\theta)\,d\theta \neq 0$, then $m = \ell + k$.

This result is next applied to study in some cases the cyclicity of the focus at the origin in the normal form for nilpotent monodromic singularities $(\dot{x}, \dot{y}) = (-y, x^{2n-1} + yb(x))$, with $b(x) = \sum_{j \geq \beta} b_j x^j$.

References


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On the approximation of periodic solutions of non-autonomous ordinary differential equations

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We recover the pioneering results of Stokes [3] and Urabe [4] that provide a theoretical basis for proving that near truncated Fourier series that approach a periodic solution of an ordinary differential equations there are actual periodic solutions of the equation. This result can be applied independently of the method that has been used to get these approximation. We will restrict our attention to one-dimensional non-autonomous ordinary differential equations and we apply the results obtained to a couple of concrete examples coming from planar autonomous systems [1]. In one of them we use the Harmonic balance method to get an approximated solution while in the other we use a numerical approach.
Dynamics at infinity and other global dynamical aspects of Shimizu-Morioka equations

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We present some global dynamical aspects of Shimizu-Morioka equations, which is a simplified model proposed for studying the dynamics of the well-known Lorenz system for large Rayleigh numbers. Using the Poincaré compactification of a polynomial vector field in $\mathbb{R}^3$, we give a complete description of the dynamics of Shimizu-Morioka equations at infinity. Then using analytical and numerical tools, we show the existence of infinitely many singularly degenerate heteroclinic cycles, each one consisting of an invariant set formed by a line of equilibria together with a heteroclinic orbit connecting two of these equilibria. The dynamical consequences of the existence of these cycles are also investigated. The present study is part of an effort aiming to describe global properties of quadratic three-dimensional vector fields with chaotic dynamical behavior, as made for instance in [1, 2, 3, 4, 5, 6, 7, 8, 9].

References


Liouville integrability and invariant algebraic curves of ordinary differential equations

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In [1] an example of an integrable Liouville planar polynomial differential system that has no finite invariant algebraic curves is provided. The present work deals with any general planar polynomial differential system, which can be written as an ordinary differential equation

\[
\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)},
\]

with \(P(x,y)\) and \(Q(x,y)\) real polynomials. We assume that this equation is Liouville integrable and determine, in terms of the degree in \(y\) of the equation, when this implies that the equation has a finite invariant algebraic curve.

References

On the dynamics of the rigid body with a fixed point: periodic orbits and integrability

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The aim of the present contribution is to study the periodic orbits of a rigid body with a fixed point and quasi–spherical shape under the effect of a Newtonian force field given by different small potentials. For studying these periodic orbits we shall use averaging theory. Moreover, we provide information on the $C^1$–integrability of these motions.

Limit Cycles for a class of continuous piecewise linear differential systems with three zones

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In this work we study the existence of limit cycles for the class of continuous piecewise linear differential systems

$$\dot{x} = X(x),$$

where $x = (x, y) \in \mathbb{R}^2$, and $X$ is a continuous piecewise linear vector field. We will consider the following situation, that we will name the three-zone case. We have two parallel straight lines $L_-$ and $L_+$ symmetric with respect to the origin dividing the phase plane in three closed regions: $R_-$, $R_0$ and $R_+$ with $(0,0) \in R_0$ and the regions $R_-$ and $R_+$ have as boundary the straight lines $L_-$ and $L_+$ respectively. We will denote by $X_-$ the vector field $X$ restrict to $R_-$, by $X_0$ the vector field $X$ restricted to $R_0$ and by $X_+$ the vector field $X$ restrict to $R_+$. We suppose that the restriction of the vector field to each one of these zones are linear systems with constant coefficients that are glued continuously at the common boundary.

We suppose the following assumptions:
(H1) $X_o$ has a real equilibrium in the interior of the region $R_o$ of focus type.

(H2) The others equilibria (real or virtual) of $X_-$ and $X_+$ are a center and a focus with different stability with respect to the focus of $X_o$.

Under these hypotheses the main result is the following.

**Theorem:** Assume that system (1) satisfies assumptions (H1) and (H2). Then system (1) has a unique limit cycle, which is hyperbolic.

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**Periodic orbits of a fourth–order non–autonomous differential equation**

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We provide sufficient conditions for the existence of periodic solutions of the fourth–order differential equation

$$u''' + qu'' + pu = \varepsilon F(t, u, u', u'', u'''),$$

where $q$, $p$ and $\varepsilon$ are real parameters, $\varepsilon$ is small and $F$ is a nonlinear non-autonomous periodic function with respect to $t$. Moreover we provide some applications.

**References**


Periodic orbits of integrable birational maps on the plane: blending dynamics and algebraic geometry, the Lyness’ case.

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A birational planar map \( F \) possessing a rational first integral, preserve a foliation of the plane given by algebraic curves, which in the case that \( F \) is not of finite order, generically is given by a foliation of elliptic curves. In this case the group structure of the elliptic foliation characterizes the dynamics of any birational map preserving it [3]. We will see how take advantage of this structure in two contexts:

1. The characterization of the set of periods appearing in the family of 2–periodic Lyness difference equations \( u_{n+2}u_n = a_n + u_{n+1} \), where \( a_n \) is a 2-cycle.
2. The negative answer to a conjecture of Zeeman about the existence of rational 9-periodic orbits of the autonomous Lyness equation \( u_{n+2}u_n = a + u_{n+1} \).

The new results presented here have been jointly obtained with G. Bastien and M. Rogalski [1], and A. Gasull and X. Xarles [2].

References


The specification property in the dynamics of linear operators

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We introduce the notion of the Specification Property (SP) for operators on Banach spaces, inspired by the usual one of Bowen for continuous maps on compact spaces. This is a very strong dynamical property related to the chaotic behaviour. Several general properties of operators with the SP are established. For instance, every operator with the SP is mixing, Devaney chaotic, and frequently hypercyclic. In the context of weighted backward shifts, the SP is equivalent to Devaney chaos. In contrast, there are Devaney chaotic operators (respectively, mixing and frequently hypercyclic operators) which do not have the SP.

Limit cycles for a class of non-linear planar piecewise-continuous vector fields

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In this work we consider a system with the form

$$\dot{z} = A_0z + \varepsilon(A(z) + \varphi_0(k, z))B,$$  \hspace{1cm} (1)

where \(z = (x, y), k, B \in \mathbb{R}^2, A_0\) is a \(2 \times 2\) matrix with eigenvalues \(\pm i\), \(A : \mathbb{R}^2 \to \mathbb{R}^2\) is given by \(A = (A_1, A_2)\), where \(A_1, A_2\) are odd degree polynomials, \(\varphi_0 : \mathbb{R} \to \mathbb{R}\) is the discontinuous function given by

$$\varphi_0(s) = \begin{cases} 
-1 & , \ s \in (-\infty, 0), \\
1 & , \ s \in (0, \infty).
\end{cases}$$

We apply the regularization of Teixeira-Sotomayor [3] to this system, obtaining a continuous system. Then we apply the Averaging Method developed in [1]. We show that the number of limit cycles of system
We remark that this work generalizes [2], where the polynomials $A_1, A_2$ are taken linear.

### References


### Periodic orbits in the Rössler prototype-4 system

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O.E. Rössler introduced several systems in the 1970s as prototypes of the simplest autonomous differential equations having chaos, the simplicity is in the sense of minimal dimension, minimal number of parameters and minimal nonlinearities.

Here we consider the Rössler prototype-4 system

\[
\begin{align*}
\dot{x} &= -y - z, \\
\dot{y} &= x, \\
\dot{z} &= \alpha y(1 - y) - \beta z,
\end{align*}
\]

introduced in [1]. See also the book [2]. This differential system exhibits chaotic motion for the parameter values around $\alpha = \beta = 1/2$, having an strange attractor. In [2] it is numerically showed that in the region of the parameter space giving by small positive values of $\alpha$ and $\beta$ there are periodic orbits of (1), see Figure 3.8 in page 69 of [2].

In this work, by using the first order averaging theory, we prove the existence of a periodic orbit of system (1) for sufficiently small positive values of the real parameters $\alpha$ and $\beta$. This confirms the numerical computations performed in earlier works. We also extend the analysis to new different parameter conditions.
Limit cycles, invariant meridians and parallels for polynomial vector fields on the torus

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We study the polynomial vector fields of arbitrary degree in $\mathbb{R}^3$ having the 2–dimensional torus

$$\mathbb{T}^2 = \{(x, y, z) \in \mathbb{R}^3 : (x^2 + y^2 - a^2)^2 + z^2 = 1\} \text{ with } a > 1,$$

invariant by their flow.

We characterize all the possible configurations of invariant meridians and parallels that these vector fields can exhibit. Furthermore we analyze when these invariant either meridians or parallels can be limit cycles.

References


Discontinuous piecewise linear differential systems with two zones in the plane: study of limit cycles

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General piecewise linear systems have been attracted great attention in the past years specially due to its simplicity. Landmark in this area is the work of Andronov et al. in [1]. Here we are interested in piecewise linear systems in the plane with two zones, that is piecewise linear systems in the plane where the two linearity regions are separated by a straight line \( \mathcal{L} \).

We study the existence of limit cycles in a one–parameter family of discontinuous piecewise linear differential systems with two zones in the plane. We prove that for suitable values of the parameter the family can have at least three limit cycles.

References


Reversibility and branching of periodic orbits

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We study the dynamics near an equilibrium point of a 2-parameter family of a reversible system in \( \mathbb{R}^6 \). In particular, we exhibit conditions for the existence of periodic orbits near the equilibrium of systems having the form \( x^{(v)} + \lambda_1 x^{(iv)} + \lambda_2 x'' + x = f(x, x', x'', x'''(iv), x(v)) \). The techniques used are Belitskii normal form combined with Lyapunov-Schmidt reduction.
Global dynamics in the Poincaré ball of the Chen system having invariant algebraic surfaces

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In this work we perform a global dynamical analysis of the Chen system

$$\dot{x} = a(y - x), \quad \dot{y} = (c - a)x - xz + cy, \quad \dot{z} = xy - bz,$$

where \((x,y,z),(a,b,c) \in \mathbb{R}^3\). This system was firstly studied in [1] and has shown to be chaotic for suitable choices of the parameters \(a, b\) and \(c\). By using the Poincaré compactification for a polynomial vector field in \(\mathbb{R}^3\) we give the complete description of its dynamics on the sphere at infinity. For six sets of the parameter values the system has invariant algebraic surfaces. In these cases we provide the global phase portrait of the Chen system and give a complete description of the \(\alpha\)– and \(\omega\)–limit sets of its orbits in the Poincaré ball, including its boundary \(S^2\), i.e. in the compactification of \(\mathbb{R}^3\) with the sphere \(S^2\) of the infinity. Moreover, we prove the existence of a family with infinitely many heteroclinic orbits contained on invariant cylinders when the Chen system has a line of singularities and a first integral, which indicates the complicated dynamical behavior of its solutions even in the absence of chaotic dynamics. Although applied to a particular case, the technique presented may be used in the global study of other polynomial systems in \(\mathbb{R}^3\).

References

Piecewise linear systems and singular perturbation techniques

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In this work we describe some qualitative and geometric aspects of piecewise linear systems on $\mathbb{R}^2$ around typical singularities. By means of a regularization process proceeded by a blow–up technique [1] we are able to establish an interaction between three important themes of the qualitative theory of non-smooth dynamical systems:

- synchronization phenomena,
- sliding vector fields (also known as Filippov systems) and
- singular perturbation.

The regularization process developed by Llibre, da Silva and Teixeira [1] is crucial for the development of this work.

References


Chaotic behaviour of operators on compact invariant sets

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In this work we will study properties such as hypercyclicity, Devaney chaos, and the transitivity of operators defined on invariant compact sets. To this end, we will focus on absolutely convex compact sets and will obtain results that will allow us to take the properties of operators defined on invariant compact sets and generalize these properties to said set’s convex hull or the Banach subspace generated by the closure of its linear hull. Moreover, we will give examples that illustrate these results and characterize when an operator defined on an invariant compact set is transitive, mixing, or chaotic.
On integrable systems on $S^3$

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A first question in the study of a flow is to know if it is integrable or not. The structure of integrable systems and the restrictions than the integrability implies has practical applications. Moreover is an area where Topology and flows are interlaced. The best known integrable systems are Hamiltonian systems. The study of two degrees of freedom Hamiltonian systems defined on a symplectic manifold is one of the areas where mathematicians and physicist has devoted a lot of efforts. Denote by $I_a(f) = \{ p \in \mathbb{R}^n | f(p) = a \}$ the level sets of $f$. In the case where $f$ is a first integral of a vector field, $I_a(f)$ is an invariant set for the flow.

An integrable hamiltonian system is a hamiltonian system with another first integral $f$ independent of $H$. One particular aspect in the study of integrable Hamiltonian systems is the decomposition of $I_a(H)$ in level sets of $f$ or in other words the study of the foliation defined on $I_a(H)$ by $f$. These foliations are singular. The study of the invariants for orbital equivalent flows and topological obstructions to the integrability are two of the main items in this area. See [1], [2], [3] and [4].

The aim of this poster is to present a generalization of the results on Hamiltonian systems to the study of systems on $S^3$ that admits a first integral that is a Morse-Bott function. Recall that $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a Morse-Bott function if its singular points are organized as non degenerate smooth critical or singular submanifolds. Here a critical submanifold of $f$ is called non degenerate if the Hessian of $f$ is non degenerate on normal planes to this submanifold.

References


Solar System transport in a chain of bicircular models

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The bicircular problem (BCP) is a simplified model for the four body problem. In this model, we assume that the Sun and Jupiter are revolving in circular orbits around their center of mass, and a planet moves in a circular orbit around their barycenter. This is not a coherent model in the sense that the trajectories of the Sun, Jupiter and the planet do not satisfy Newton’s equations. In a first step, our aim is to consider the Solar System as a set of coupled bicircular models. This is done in order to obtain a first insight of transport in the Solar System that may be explained using the separated bicircular problems. The invariant manifolds of convenient periodic orbits of each particular bicircular problem are considered in order to find connections between two consecutive problems. These connections allow to obtain a mechanism to explain transport of infinitesimal particles towards the inner Solar System. Finally the results obtained are validated using the complete Solar System.
Quasi-homogeneous planar polynomial differential systems and their integrability

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The planar differential system \( \dot{x} = P(x,y), \dot{y} = Q(x,y) \), with \( P,Q \in \mathbb{C}[x,y] \) is quasi–homogeneous (abreviately, QH-system) if there exist \( s_1, s_2, d \in \mathbb{N} \) such that for any arbitrary \( \alpha \in \mathbb{R}^+ \), \( P(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_1-1+d}P(x,y) \), \( Q(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_2-1+d}Q(x,y) \). The QH-systems have been studied from many different points of view (integrability, centers, normal forms, limit cycles (see, for example, [1], [2], [3]). But so far, there was not an algorithm for constructing all the QH-systems of a given degree. In this work we provide such an algorithm and, as an application, we obtain all QH-systems of degree 2 and 3. Moreover, since this QH-systems are Liouvillian integrable, we characterize all the QH-systems of degree 2 and 3 having a polynomial, rational or global analytical first integral.

References


Existence and uniqueness of limit cycles for generalized φ-laplacian Liénard equations

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Liénard equation,

\[ x'' + f(x)x' + g(x) = 0, \]

appears as a simplified model in many problems of science and engineering. Since the first half of 20th century, many papers have appeared giving existence and uniqueness conditions for the limit cycles that a Liénard equation exhibits.

In [1], we extend some of these results for the case of the generalized φ-laplacian Liénard equation

\[ (\varphi(x'))' + f(x)\psi(x') + g(x) = 0. \]

This generalization appears when other derivations, different from the classic one, are considered, such as the relativistic one. Our results apply, for example, to the relativistic van der Pol equation

\[ \left( \frac{x'}{\sqrt{1 - \frac{x^2}{c^2}}} \right)' + \mu(x^2 - 1)x' + x = 0. \]

References

On the reversible quadratic polynomial vector fields on the two dimensional sphere

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We study a class of quadratic reversible polynomial vector fields on $S^2$ with $(3, 2)$-type reversibility. We classify all isolated singularities and we prove the nonexistence of limit cycles for this class. Our study provides tools to determine the phase portrait for these vector fields.

References


Number of invariant straight lines for homogeneous polynomial vector fields of arbitrary degree and dimension

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We study the number of invariant straight lines through the origin of the homogeneous polynomial differential systems of degree $m$ in $\mathbb{R}^d$ or $\mathbb{C}^d$, when this number is finite. This notion extends in the
natural way the classical notion of eigenvectors of homogeneous linear differential systems to homogeneous polynomial differential systems. This number provides an upper bound for the number of infinite singular points of the polynomial differential systems of degree \(m\) in \(\mathbb{R}^d\). This upper bound is reached if all the invariant straight lines through the origin are real.

Rotation intervals for quasi-periodically forced circle maps

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Rotations numbers have been long used to describe the dynamics of circle maps. It is known that for invertible maps the rotation number is independent of the initial point, whereas for non-invertible maps different points may have different rotation rates and thus the dynamics can be more complex. Nevertheless, rotation numbers for a given choice of parameters always form a closed interval (including the possibility of a point).

Analogous results have been proved for quasi-periodically forced circle maps [1], however, the structure in terms of rotation numbers in several regions of the parameter space remains unknown.

This piece of work computes the borders of rotation intervals using an extension to the quasi-periodically forced case of Boyland’s method [2], which relies in the construction of an associated family of invertible maps, and the algorithms on [1], specifically in the case of a large coupling force.

These results show that for large coupling strength, the boundaries of the rotation interval are approximated by integrating the maps on Boyland’s construction. As a result, the length of the rotation interval can be approximated, and a scaling for its growth is presented [3].

References


Phase portraits on the Poincaré disc of a SIS model

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In the qualitative theory of ordinary differential equations, we can find many papers whose propose is the classification of all the possible topological phase portraits of a given family of differential system. Most of the studies rely on systems with real parameters and the study consists of outlining their phase portraits by finding out some conditions on the parameters. Here, we studied a susceptible-infectious-susceptible (SIS) model given by
\[
\dot{x} = -bxy - mx + cy + mk, \quad \dot{y} = bxy - (m + c)y,
\]
where \(b, c, k, m\) are real parameters with \(b \neq 0, m \neq 0\) [1]. Such system describes an infectious disease from which infected people recover with immunity against reinfection. The integrability of such system has already been studied by Nucci and Leach [3] and Llibre and Valls [2]. We found out two different topological classes of phase portraits.

References


New doubly-symmetric families of comet like periodic orbits in the spatial restricted (N+1)–body problem

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For any positive integer \(N \geq 2\) we prove the existence of a new family of periodic solutions for the spatial restricted \((N+1)\)–body problem. In these solutions the infinitesimal particle is very far from the primaries, have large inclinations and have some symmetries. In fact we extend results of Howison and Meyer, see \([1]\), from \(N = 2\) to any positive integer \(N \geq 2\).

References


Firing map for periodically and almost-periodically driven integrate-and-fire models: a dynamical systems approach

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We consider the so-called integrate-and-fire system, in which a continuous dynamics induced by the differential equation \(\dot{x} = f(t, x), f : \mathbb{R}^2 \to \mathbb{R}\), is interrupted by the threshold and reset behaviour: \(\lim_{t \to s^+} x(t) = x_r\) if \(x(s) = x_{tr}\). The question is to describe the sequence of consecutive resets \(s_n\) for a trajectory starting at time \(t_0\) from the resting value \(x_r\) as iterations of some map \(\Phi^n(t_0) = s_n\), called the firing map, and the sequence of interspike-intervals \(\eta_n(t) = s_n - s_{n-1}\) as a sequence of displacements \(\Phi^n(t_0) - \Phi^{n-1}(t_0)\) along a trajectory of this map. The problem appears in various applications, for example in modelling of an action potential (spiking) by a neuron \(([1, 2, 3])\).

We investigate behaviour of the sequence of interspike-intervals when the function \(f(t, x)\) is smooth enough and periodic in \(t\). In this case the problem is covered by analysis of the displacement sequence of an orientation preserving homeomorphism (diffeomorphism) \(\varphi\) of the circle \(([4])\), which is a projection of the firing map \(\Phi\) onto \(S^1\). If the firing rate, which is the rotation number \(\varrho(\Phi)\), is rational, say \(\varrho(\Phi) = p/q\), then \(\eta_n(t)\) is asymptotically periodic with frequency \(q\). If \(\varrho(\Phi) \notin \mathbb{Q}\), then the values of \(\eta_n(t)\) are dense in a set which depends on the map \(\gamma\) (semi-) conjugating the firing phase map \(\varphi\) with the rotation by \(\varrho\)
and which is the support of the displacements distribution with respect to the invariant measure of \( \varphi \).

Further, with the use of topological dynamics, we discuss the recurrent properties of the sequence \( \eta_n(t) \).

We show how these results are reflected by interspike-intervals in particular integrate-and-fire models.

However, in view of applications we shall also weaken the assumption that the input function is periodic and continuous. It turns out that many of the required properties of the firing map for most commonly used models, Perfect Integrator \( \dot{x} = f(t) \) and Leaky Integrate-and-Fire \( \dot{x} = -\sigma x + f(t) \), still hold if \( f \) is only locally integrable and almost periodic, either uniformly or in a sense of Stepanov ([5]).

References


An empirical stability analysis of the Caledonian symmetric four-body model

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The Caledonian Symmetric Four Body Problem (CSFBP) is a restricted four body system with a symmetrically reduced phase space which can be applied to study the stability and evolution of symmetric...
quadruple stellar clusters and exo-planetary systems [1]. Recently we have developed a global regularization scheme that consists of adapted versions of several known regularization transformations such as the Levi-Civita-type coordinate transformations; that together with a time transformation, removes all the singularities due to colliding pairs of masses [2]. Using this newly developed numerical algorithm, we numerically investigate the relationship between the hierarchical stability of the system and the analytical stability parameter characterised by the Szebehely constant which is a function of the total energy and angular momentum of the system. It is possible to empirically analyse the stability of the CSFBP by studying the hierarchical evolution of a comprehensive set of orbits appearing in the phase space of the CSFBP.

References


Regularization and singular perturbation techniques for non-smooth systems

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This work is concerned with some aspects of the qualitative-geometric theory of Non-smooth Systems. We present a connection between the regularization process of non-smooth vector fields and the singular perturbation problems. As a matter of fact, the main results in our setting fill a gap between these areas. We give special attention for singular perturbation problems which present discontinuous slow flow. More specifically we consider the following system.

\[
\begin{align*}
\dot{x} &= \begin{cases} 
F(x, y, \varepsilon) & \text{if } h(x, y, \varepsilon) \leq 0, \\
G(x, y, \varepsilon) & \text{if } h(x, y, \varepsilon) \geq 0, 
\end{cases} \\
\varepsilon \dot{y} &= H(x, y, \varepsilon),
\end{align*}
\]

where \( \varepsilon \in \mathbb{R} \) is a positive small parameter, \( x = (x_1, \ldots, x_n) \in \mathbb{R}^n \) and \( y \in \mathbb{R} \) denote the slow and fast variables, respectively, and \( F = (F_1, \ldots, F_n), G = (G_1, \ldots, G_n), h \) and \( H \) are \( C^r \) maps which vary differentially with respect to \( \varepsilon \). We assume that \( r \) is big enough for our purposes. More specifically, our main question is to know how the dynamics of Filippov systems is affected by singular perturbations. We extend the Fenichel Theory developed to these systems.
Global dynamics of the Kummer–Schwarz differential equation

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In this talk/poster we present the study of the Kummer–Schwarz differential equation \(2\ddot{x} - 3\dot{x}^2 = 0\) which is of special interest due to its relationship with the Schwarzian derivative. This differential equation is transformed into a first order differential system in \(\mathbb{R}^3\), and we provide a complete description of its global dynamics adding the infinity.

References


Totally ordered non-singular Morse-Smale flows on $S^3$

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Non-singular Morse-Smale flows are characterized by the round handle decomposition of the manifold where they are defined [3], [1]. For NMS flows on the 3-dimensional sphere $S^3$, M. Wada obtains a characterization of the flows in terms of links of periodic orbits [4].

From the round handle decomposition of NMS flows on $S^3$ we determine which flows have heteroclinic trajectories due to transversal intersections of invariant manifolds [2].

In this paper we show that the presence of heteroclinic trajectories imposes an order in the round handle decomposition of a Nonsingular Morse-Smale flow on $S^3$. We also obtain that this order is total for NMS flows composed of one repulsive, one attractive and $n$ unknotted saddle orbits.

References


The existence of traveling wave of FitzHugh-Nagumo system

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In this paper, considering traveling wave of FitzHugh-Nagumo system, we have to study a three-dimensional nonlinear ordinary differential equations. By analyzing the higher-dimension Hopf bifurcation, we show that there exist three small amplitude periodic traveling wave of FitzHugh-Nagumo System with some parameters $0 < a < 1/2, b > 0, d > 0$. 
A particular family of globally periodic birational maps

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In [1] the authors make a classification of a family of birational surface maps. They have identified the subfamilies with dynamical degree $1 \leq D \leq 2$, where

$$D = \lim_{n \to \infty} (\deg(f^n))^\frac{1}{n},$$

see [3]. We investigate one of the subfamilies of [1] with $D = 1$. For complex numbers $\alpha, \beta$ and $\gamma$, we consider the following family of birational maps:

$$f(x, y) = \left(\alpha x, \frac{\beta + x}{\gamma + y}\right).$$

Let $F : \mathbb{P}^2 \to \mathbb{P}^2$ be the extension of $f$ in projective space. We define the indeterminacy locus of $F$ as $I = \{O_0, O_1, O_2\}$. In order to regularize $F$ we blow up all the orbits of points under $F$ that reach any indeterminacy point of $F$. Trivially there are two orbits of points $A_0$ and $A_1$ that reach $O_2$ and $O_1$ respectively. We then impose the condition on the orbit of $A_2$ such that $F^p(A_2) = O_0$, where $A_0, A_1, A_2$ are indeterminacy points of $F^{-1}$. Let $X$ be the space we get after blowing up operation and let $\tilde{F}$ be the induced map. We find that $\tilde{F}$ is an algebraically stable map which induces a morphism of groups $F^*: \text{Pic}(X) \to \text{Pic}(X)$. This map $F^*$ is a homeomorphism on $\text{Pic}(X)$. We claim that $F$ is a globally periodic map with period $2p + 2 \forall p \in \mathbb{N}$ provided that $F^p(A_2) = O_0$. We prove this by first showing that $(F^*)^{2p+2}$ is the identity map and then we show that $F^{2p+2}$ is a linear map, by using a result from [5]. Finally by using previous results we prove that $(F)^{2p+2}$ is the identity map.

References


Computing analytically periodic orbits of differential equations

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We provide an analytic algorithm for computing periodic orbits of differential equations in dimension \( n \geq 2 \) having an equilibrium point with eigenvalues \( \pm \omega i \) and \( \rho_k \) with \( \omega \neq 0 \) and if \( n \geq 3 \) then \( \rho_k \in \mathbb{R} \) for \( k = 3, \ldots, n \). Moreover, our method needs that when we translate the equilibrium point at the origin of coordinates, the non–linear part of the translated differential equation depends on a multiplicative small parameter. We provide two applications of this algorithm, one related with a 3–dimensional differential equation of L. Chua’s class, and the other to a 5–dimensional differential equation due to E.N. Lorenz.

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