Dynamics of trace maps motivated by applications in spectral theory of quasicrystals

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Since the discovery of quasicrystals by Schehtman et. al. in the early eighties, quasiperiodic models in mathematical physics have formed an active area of research. In the pioneering works of M. Kohmoto et. al. [1] and M. Casdagli [2], a strong relationship between trace maps and spectral properties of quasiperiodic Schrödinger operators was discovered.

We discuss the dynamics of the so-called Fibonacci trace map, associated to a prototypical quasiperiodic model, and demonstrate how it can be applied in the study of spectral properties of a class of quasiperiodic operators. The Fibonacci trace map is an analytic map on the three-dimensional Euclidean space exhibiting nontrivial behavior (hyperbolicity on some invariant two-dimensional surfaces and partial hyperbolicity on a three-dimensional submanifold of $\mathbb{R}^3$ foliated by these invariant surfaces). This in turn has strong implications in spectral theory of the associated quasiperiodic quantum Hamiltonians (discrete Schrödinger and Jacobi operators on $\ell^2(\mathbb{C})$), such as fractal structure of the operator spectrum, estimates on fractal dimensions, regularity of fractal dimensions and the like [3, 4, 5, 6, 7, 8].

References

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