Extending geometric singular perturbation theory for ordinary differential equations with three time–scales

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Systems in nature, which are modeled by ordinary differential equations, often involve two or more different time scales. For instance, in biological literature we can find many examples of models which present such features. When systems present a clear separation in time scales, methods of approximations of slow–fast systems can be applied. Around 1980, geometric singular perturbation theory was introduced. The foundation of this theory was laid by Fenichel [1] and it essentially uses geometric methods from dynamical systems theory for studying the properties of solutions of the system. Note that for the singular perturbation problems studied by Fenichel [1] only two different time–scales can be derived: a slow and a fast ones.

In this poster we consider systems of ordinary differential equations involving three different time–scales. These systems are in general written in the form

$$\varepsilon x' = f(\mathbf{x}, \varepsilon, \delta), \qquad y' = g(\mathbf{x}, \varepsilon, \delta), \qquad z' = \delta h(\mathbf{x}, \varepsilon, \delta), \tag{1}$$

where $\mathbf{x} = (x, y, z) \in \mathbb{R}^3$, ε and δ are two independent small parameter ($0 < \varepsilon,\ \delta \ll 1$), and f, g, h are C^r functions, where r is big enough for our purposes. Now, three different time–scales can be derived of the system (1): a *slow* time–scale t, an *intermediate* time–scale $\tau_1 := \frac{t}{\delta}$ and a *fast* time–scale $\tau_2 := \frac{\tau_1}{\varepsilon}$.

We intend to develop a geometric theory, similar the one given by Fenichel [1], for systems of the form (1). We present in this poster some results in this direction.

References

[1] Neil Fenichel, *Geometric singular perturbation theory for ordinary differential equations*, J. Diff. Equations **31** (1979), 53–98.