Regularization and singular perturbation techniques for non-smooth systems

Paulo Ricardo da Silva

Universidade Estadual Paulista, Rua C. Colombo, 2265, CEP 15054–000, S. J. Rio Preto, São Paulo, Brazil
E-mail: `prs@ibilce.unesp.br`
URL: `http://www.mat.ibilce.unesp.br/sisdin/`

This work is concerned with some aspects of the qualitative-geometric theory of Non-smooth Systems. We present a connection between the regularization process of non-smooth vector fields and the singular perturbation problems. As a matter of fact, the main results in our setting fill a gap between these areas. We give special attention for singular perturbation problems which present discontinuous slow flow. More specifically we consider the following system.

$$\begin{aligned} \dot{x} &= \begin{cases} F(x,y,\varepsilon) & \text{if} \quad h(x,y,\varepsilon) \leq 0, \\ G(x,y,\varepsilon) & \text{if} \quad h(x,y,\varepsilon) \geq 0, \end{cases} \\ \varepsilon \dot{y} &= H(x,y,\varepsilon), \end{aligned} \tag{1}$$

where $\varepsilon \in \mathbb{R}$ is a positive small parameter, $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $y \in \mathbb{R}$ denote the slow and fast variables, respectively, and $F = (F_1, \dots, F_n)$, $G = (G_1, \dots, G_n)$, h and H are C^r maps which vary differentially with respect to ε. We assume that r is big enough for our purposes. More specifically, our main question is to know how the dynamics of Filippov systems is affected by singular perturbations. We extend the Fenichel Theory developed to these systems.