

The global topological classification of the Lotka-Volterra quadratic differential systems

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The planar Lotka-Volterra systems intervene in many areas of applied mathematics, so naturally there were several attempts to give the topological classification of these systems, the last one published in 2008. These classifications are far from satisfying because apart from containing errors or being incomplete, they are done without the use of adequate global tools and so we end up with a maze of tables containing numerous cases expressed in terms of inequalities of the coefficients of the systems which fail to convey to us the global phenomena involved. In this work, jointly done with Nicolae Vulpe, we base our classification on the global concept of *configuration of invariant lines* of the systems. After a first work classifying the systems in terms of their associated configurations of invariant lines, in the present work we take each class determined by a specific configuration and classify it topologically. The final result is stated in terms of algebraic invariants. We give necessary and sufficient conditions in terms of invariant polynomials, which can be computed using computer algebra, for obtaining each one of the specific phase portraits of this class.

References

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