

Hunting three nested limit cycles with only two linear foci

EMILIO FREIRE¹, ENRIQUE PONCE², FRANCISCO TORRES³

¹ *Departamento of Matemática Aplicada II, Escuela Técnica Superior de Ingeniería, Camino de ls Descubrimientos, 41092 Sevilla, Spain.*

E-mail: efrem@us.es

² *Departamento of Matemática Aplicada II, Escuela Técnica Superior de Ingeniería, Camino de ls Descubrimientos, 41092 Sevilla, Spain.*

E-mail: eponcem@us.es

³ *Departamento of Matemática Aplicada II, Escuela Técnica Superior de Ingeniería, Camino de ls Descubrimientos, 41092 Sevilla, Spain.*

E-mail: ftorres@us.es

When the aggregation of two linear differential systems defines a discontinuous planar vector field, the problem of determining the maximum number of nested limit cycles surrounding only one equilibrium point, is more challenging than in the continuous case, where it is possible to establish the existence at most of one limit cycle, see [1]. In fact, by considering a specific family of discontinuous differential systems with two linear zones sharing the equilibrium position, strong numerical evidence about the existence of three nested limit cycles was obtained very recently in [3], contrarily to what it had been conjectured in [4]. The example in [3] has a real unstable focus and a virtual stable focus sharing their location. A rigorous, computer assisted proof of the existence of such limit cycles has been obtained in [5], but some explanation on their generation mechanism was lacking.

We will show, thanks to the canonical forms introduced in [2], how to analytically prove the existence of such three limit cycles in more general cases, by combining adequately the two linear foci. The hunting of these limit cycles can be done by using two different approaches, both in a bifurcation spirit. We can perturb a crossing-sliding limit cycle that coexists with a non-hyperbolic periodic orbit at infinity, but also the three limit cycles can bifurcate from a higher degeneration at infinity. We will mainly pay attention to the first mechanism.

References

- [1] E. Freire, E. Ponce, F. Rodrigo, and F. Torres, *Bifurcation sets of continuous piecewise linear systems with two zones*, *Int. J. Bifurcation and Chaos* **8** (1998), 2073–2097.
- [2] E. Freire, E. Ponce, and F. Torres, *Canonical Discontinuous Planar Piecewise Linear Systems*, *SIAM J. Applied Dynamical Systems* **11** (2012), 181–211.
- [3] S.-M. Huan, and X.-S. Yang, *On the number of limit cycles in general planar piecewise linear systems*, *Discrete and Continuous Dynamical Systems-A* **32** (2012), 2147–2164.
- [4] M. Han, and W. Zhang, *On Hopf bifurcation in non-smooth planar systems*, *J. of Differential Equations* **248** (2010), 2399–2416.
- [5] Llibre, J., and E. Ponce, *Three nested limit cycles in discontinuous piecewise linear differential systems with two zones*, *Dynamics of Continuous, Discrete & Impulsive Systems-B: Applications & Algorithms* **19** (2012) 325–335.