Quasi-homogeneous planar polynomial differential systems and their integrability

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The planar differential system $\dot{x} = P(x, y), \dot{y} = Q(x, y)$, with $P, Q \in \mathbb{C}[x, y]$ is *quasi–homogeneous* (abreviately, QH-system) if there exist $s_1,\ s_2, d \in \mathbb{N}$ such that for any arbitrary $\alpha \in \mathbb{R}^+$, $P(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_1-1+d}P(x, y)$, $\quad Q(\alpha^{s_1}x, \alpha^{s_2}y) = \alpha^{s_2-1+d}Q(x, y)$. The QH-systems have been studied from many different points of view (integrability, centers, normal forms, limit cycles (see, for example, [1], [2], [3]). But so far, there was not an algorithm for constructing all the QH-systems of a given degree. In this work we provide such an algorithm and, as an application, we obtain all QH-systems of degree 2 and 3. Moreover, since this QH-systems are Liouvillian integrable, we characterize all the QH-systems of degree 2 and 3 having a polynomial, rational or global analytical first integral.

References

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