

On the Darboux theory of integrability of non-autonomous polynomial differential systems

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In this work we unfold some differential algebraic aspects of Darboux first integrals of polynomial vector fields [1, 4]. An interesting improvement is that our approach can be applied both to autonomous and non-autonomous vector fields. We give a sufficient and necessary condition for the existence of a Darboux first integral of a specific form for a polynomial vector field with some known algebraic invariant hypersurfaces. For the autonomous case, the classical result of Darboux is obtained as a corollary [3]. For the non-autonomous case our characterization improves a known criterium of Llibre and Pantazi, [2].

References

- [1] G. Darboux, *Mémoire sur les équations différentielles algébriques du premier ordre et du premier degré (Mélanges)*, Bull. Sci. math. 2ème série **2** (1878), 60–96; 123–144; 151–200.
- [2] J. Llibre and Ch. Pantazi, *Darboux theory of integrability for a class of nonautonomous vector fields*, J. Math. Phys., **50**, (2009) 102705.
- [3] J. Llibre and X. Zhang, *Rational first integrals in the Darboux theory of integrability in C^n* , Bull. Sci. Math. **134** (2010), no. 2, 189–195.
- [4] M.F. Singer, *Liouvillian first integrals of differential equations*, Trans. Amer. Math. Soc. **333** (1992) 673–688.