On integrable systems on $\mathbb{S}^3$

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A first question in the study of a flow is to know if it is integrable or not. The structure of integrable systems and the restrictions than the integrability implies has practical applications. Moreover is an area where Topology and flows are interlaced. The best known integrable systems are Hamiltonian systems. The study of two degrees of freedom Hamiltonian systems defined on a symplectic manifold is one of the areas where mathematicians and physicist has devoted a lot of efforts. Denote by $I_a(f) = \{p \in \mathbb{M}^n |\, f(p) = a\}$ the level sets of f. In the case where f is a first integral of a vector field, $I_a(f)$ is an invariant set for the flow.

An integrable hamiltonian system is a hamiltonian system with another first integral f independent of H. One particular aspect in the study of integrable Hamiltonian systems is the decomposition of $I_a(H)$ in level sets of f or in other words the study of the foliation defined on $I_a(H)$ by f. These foliations are singular. The study of the invariants for orbital equivalent flows and topological obstructions to the integrability are two of the main items in this area. See [1], [2], [3] and [4].

The aim of this poster is to present a generalization of the results on Hamiltonian systems to the study of systems on $\mathbb{S}^3$ that admits a first integral that is a Morse-Bott function. Recall that $f : \mathbb{M}^n \to \mathbb{R}$ is a Morse-Bott function if its singular points are organized as non degenerate smooth critical or singular submanifolds. Here a critical submanifold of f is called non degenerate if the Hessian of f is non degenerate on normal planes to this submanifold.

References

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