Global dynamics in the Poincaré ball of the Chen system having invariant algebraic surfaces

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In this work we perform a global dynamical analysis of the Chen system

$$\dot{x} = a(y - x), \quad \dot{y} = (c - a)x - xz + cy, \quad \dot{z} = xy - bz,$$

where $(x, y, z), (a, b, c) \in \mathbb{R}^3$. This system was firstly studied in [1] and has shown to be chaotic for suitable choices of the parameters a, b and c. By using the Poincaré compactification for a polynomial vector field in $\mathbb{R}^3$ we give the complete description of its dynamics on the sphere at infinity. For six sets of the parameter values the system has invariant algebraic surfaces. In these cases we provide the global phase portrait of the Chen system and give a complete description of the α– and ω–limit sets of its orbits in the Poincaré ball, including its boundary $\mathbb{S}^2$, i.e. in the compactification of $\mathbb{R}^3$ with the sphere $\mathbb{S}^2$ of the infinity. Moreover, we prove the existence of a family with infinitely many heteroclinic orbits contained on invariant cylinders when the Chen system has a line of singularities and a first integral, which indicates the complicated dynamical behavior of its solutions even in the absence of chaotic dynamics. Although applied to a particular case, the technique presented may be used in the global study of other polynomial systems in $\mathbb{R}^3$.

References

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