Limit cycles for a class of non-linear planar piecewise-continuous vector fields

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In this work we consider a system with the form

$$\dot{\mathbf{z}} \;=\; A_0\mathbf{z} + \varepsilon(A(\mathbf{z}) + \varphi_0(\langle \mathbf{k}, \mathbf{z}\rangle)B), \tag{1}$$

where $\mathbf{z} = (x, y)$, $\mathbf{k}, B \in \mathbb{R}^2$, A_0 is a 2×2 matrix with eigenvalues $\pm i$, $A : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $A = (A_1, A_2)$, where A_1, A_2 are odd degree polynomials, 3. $\varphi_0 : \mathbb{R} \to \mathbb{R}$ is the discontinuous function given by

$$\varphi_0(s) = \begin{cases} -1 & , \quad s \in (-\infty, 0), \\ 1 & , \quad s \in (0, \infty). \end{cases}$$

We apply the regularization of Teixeira-Sotomayor [3] to this system, obtaining a continuous system. Then we apply the Averaging Method developed in [1]. We show that the number of limit cycles of system 1 depends on the degree of the polinomials A_1, A_2. We also determine upper bounds for the number of limit cycles.

We remark that this work generalizes [2], where the polynomials A_1, A_2 are taken linear.

References

[1] A. Buica, J. Llibre. *Averaging methods for finding periodic orbits via Brouwer degree.* Bull. Sci. Math. 128 (2004), 7–22.

[2] M. F. Silva Lima, J. Llibre. *Limit cycles and invariant cylinders for a class of continuous and discontinuous vector field in dimention 2n.* Applied Mathematics and Computation 217 (2011), 9985–9996.

[3] J. Sotomayor, M. A. Teixeira. *Regularization of discontinuous vector fields.* International Conference on Differential Equations (Lisboa, 1995), 207–223, World Sci. Publ., River Edge, NJ, 1998.