Periodic orbits of integrable birational maps on the plane: blending dynamics and algebraic geometry, the Lyness' case.

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A birational planar map F possessing a rational first integral, preserve a foliation of the plane given by algebraic curves, which in the case that F is not of finite order, generically is given by a foliation of elliptic curves. In this case the group structure of the elliptic foliation characterizes the dynamics of any birational map preserving it [3]. We will see how take advantage of this structure in two contexts:

(1) The characterization of the set of periods appearing in the family of 2–periodic Lyness difference equations $u_{n+2}u_n = a_n + u_{n+1}$, where a_n is a 2-cycle.

(2) The negative answer to a conjecture of Zeeman about the existence of rational 9-periodic orbits of the autonomous Lyness equation $u_{n+2}u_n = a + u_{n+1}$.

The new results presented here have been jointly obtained with G. Bastien and M. Rogalski [1], and A. Gasull and X. Xarles [2].

References

[1] G. Bastien, V. Mañosa, M. Rogalski, *On the periodic solutions of* 2-*periodic Lyness difference equations.* Preprint, arXiv:1201.1027v1 [math.DS]

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[3] D. Jogia, J.A.G. Roberts, F. Vivaldi, *An algebraic geometric approach to integrable maps of the plane,* Journal of Physics A, **39** (2006), 1133–1149