Limit Cycles for a class of continuous piecewise linear differential systems with three zones

Mauricio F.S. Lima[1], Jaume Llibre[2]

[1] *Centro de Matemática Computação e Cognição. Universidade Federal do ABC, 09210-170. Santo André. S.P. Brazil.*
E-mail: mauricio.lima@ufabc.edu.br

[2] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain.*
E-mail: jllibre@mat.uab.cat

In this work we study the existence of limit cycles for the class of continuous piecewise linear differential systems

$$\mathbf{x}' = X(\mathbf{x}), \tag{1}$$

where $\mathbf{x} = (x, y) \in \mathbb{R}^2$, and X is a continuous piecewise linear vector field. We will consider the following situation, that we will name the three-zone case. We have two parallel straight lines L_- and L_+ symmetric with respect to the origin dividing the phase plane in three closed regions: R_-, R_o and R_+ with $(0, 0) \in R_o$ and the regions R_- and R_+ have as boundary the straight lines L_- and L_+ respectively. We will denote by X_- the vector field X restrict to R_-, by X_o the vector field X restricted to R_o and by X_+ the vector field X restrict to R_+. We suppose that the restriction of the vector field to each one of these zones are linear systems with constant coefficients that are glued continuously at the common boundary.

We suppose the following assumptions:

(H1) X_o has a real equilibrium in the interior of the region R_o of focus type.

(H2) The others equilibria (real or virtual) of X_- and X_+ are a center and a focus with different stability with respect to the focus of X_o.

Under these hipotesys the main result is the following.

Theorem: Assume that system (1) satisfies assumptions (H1) and (H2). Then system (1) has a unique limit cycle, which is hyperbolic.