

Uniqueness of limit cycles for Liénard differential equations of degree four

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A. Lins, W. de Melo and C. C. Pugh [4] conjectured that the classical Liénard differential equation of degree n has at most $\lfloor \frac{n-1}{2} \rfloor$ limit cycles, and they proved that the conjecture is true for $n = 3$. F. Dumortier, D. Panazzolo and R. Roussarie [2] gave a counterexample to this conjecture for $n = 7$ and they mentioned that it can be extended to $n \geq 7$ odd. Recently, P. De Maesschalck and F. Dumortier [1] proved that the classical Liénard differential equation of degree $n \geq 6$ can have $\lfloor \frac{n-1}{2} \rfloor + 2$ limit cycles. Xianwu Zeng [5] found a sufficient condition to guarantee the uniqueness of limit cycles for a subclass of classical Liénard differential equations of degree four.

In the talk we introduce a recent result [3] that any classical Liénard differential equation of degree four has at most one limit cycle, and the limit cycle is hyperbolic if it exists. This gives a positive answer to the above conjecture for $n = 4$.

References

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