Synchronisation predictions via extended phase response curves

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The phase of an oscillator such as a spiking neuron is one of the main indicators of the effects of external stimuli on the (membrane potential) dynamics. Experimentally, the phase advancement is mostly computed through phase response curves (PRCs) obtained from recordings of the time variations in reaching the next peak of the membrane potential; successful methods have been used to predict it by means of theoretical PRCs evaluated on the attractor (limit cycle). However, stimulation in transient states may induce phase advancements that differ from the predictions given in the asymptotic state. By computing the isochrons (curves of constant phase) in a vicinity of the limit cycle, we are able to accurately generalize the PRCs to the transient states and, as well, to provide a methodology to compute the phase advancement under any type of stimulus (weak or strong, instantaneous or long-lasting). In this communication, we would like to emphasize the combination of different dynamical systems approaches to an applied problem: our first inspiration being the application of Lie symmetries for time control both in planar centers and around limit cycles (see [2] and [3]), we have ended up by studying phase advancement in oscillators (see [4]) using geometric theory of invariant manifolds (see [1]). We will finish by illustrating the implications of our results to synchrony prediction in systems under high-frequency periodic stimuli by means of the study of rotation numbers for 2D maps derived from the extended PRCs. [5])

References

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