

Dynamics at infinity and other global dynamical aspects of Shimizu-Morioka equations

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We present some global dynamical aspects of Shimizu-Morioka equations, which is a simplified model proposed for studying the dynamics of the well-known Lorenz system for large Rayleigh numbers. Using the Poincaré compactification of a polynomial vector field in R^3 , we give a complete description of the dynamics of Shimizu-Morioka equations at infinity. Then using analytical and numerical tools, we show the existence of infinitely many singularly degenerate heteroclinic cycles, each one consisting of an invariant set formed by a line of equilibria together with a heteroclinic orbit connecting two of these equilibria. The dynamical consequences of the existence of these cycles are also investigated. The present study is part of an effort aiming to describe global properties of quadratic three-dimensional vector fields with chaotic dynamical behavior, as made for instance in [1, 2, 3, 4, 5, 6, 7, 8, 9].

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