

Symplectic surface diffeomorphisms

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Suppose M is a compact oriented surface of genus 0. We establish a structure theorem for area preserving diffeomorphisms of M with zero entropy and at least three periodic points. As an application we show that rotation number is defined and continuous at every point of a zero entropy area preserving diffeomorphism of the annulus.

Further applications give insight into the algebraic structure of $\text{Symp}_\mu^\omega(M)$, the group of analytic symplectic diffeomorphisms of M . We show that if G is a subgroup of $\text{Symp}_\mu^\omega(M)$ which has an infinite normal solvable subgroup, then G is virtually abelian. In particular the centralizer $\text{Cent}(f)$ of an infinite order $f \in \text{Symp}_\mu^\omega(M)$ is virtually abelian. Another immediate corollary is that if G is a solvable subgroup of $\text{Symp}_\mu^\omega(M)$ then G is virtually abelian.