Some new results on Darboux integrable differential systems

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We deal with differential systems X of the form $\dot{x} = P(x,y)$, $\dot{y} = Q(x,y)$ of degree d having a *Darboux first integral* H and an *inverse integrating factor* V. Our first main result compares a natural extension of the degree of V with $d+1$.

Theorem 1. *Let* $\Pi_1 = \prod_{i=1}^{p} \tilde{f}_i^{\lambda_i}$, $\Pi_2 = \tilde{g}/\prod_{i=1}^{p} \tilde{f}_i^{n_i}$. *Let* $\delta\left(\prod g_i^{\alpha_i}\right) = \sum \alpha_i \deg g_i$.

(a) $\delta(V) < d+1$ *if and only if* $\delta(\Pi_2) > 0$.

(b) $\delta(V) = d+1$ *if and only if either* $\delta(\Pi_2) < 0$ *and* Π_1 *is not constant, or* $\delta(\Pi_2) = 0$.

(c) $\delta(V) > d+1$ *if and only if* $\delta(\Pi_2) < 0$ *and* Π_1 *is constant.*

Moreover in all cases we have an expression of the characteristic polynomial in terms of some inverse integrating factor.

Corollary. *The infinity is degenerate if and only if* $\delta(\Pi_1) = 0$ *and either* $\delta(\Pi_2) < 0$ *and* Π_1 *is not constant, or* $\delta(\Pi_2) = 0$.

The remarkable values and remarkable curves of rational first integrals were first introduced by Poincaré and afterwards studied by several authors. It has been shown in the literature that the remarkable curves play an important role in the phase portrait as they are strongly related to the separatrices.

In our work we first define remarkable values and remarkable curves of Darboux first integrals and afterwards we state the following result.

Theorem 2. *Suppose that system X has a Darboux first integral H which is not rational. Then V is a polynomial if and only if H has no critical remarkable values.*