Global Dynamics of the Lev Ginzburg Equation

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In this poster we present the global dynamics of the first order planar polynomial differential system of degree 2, called Lev Ginzburg differential system,

$$\begin{aligned} x' &= \frac{dx}{dt} = y, \\ y' &= \frac{dy}{dt} = (1 - \beta_1 y)(\gamma - \alpha x + \beta y), \end{aligned} \tag{1}$$

depending on four parameters: $\alpha > 0$, $\beta_1 > 0$, $\gamma > 0$ and $\beta \in \mathbb{R}$.

Bellamy and Mickens [5] claimed that the Lev Ginzburg differential equation (1) can exhibit a limit cycle coming from a Hopf bifurcation. In [2] the authors shown that this differential equation has neither a Hopf bifurcation, nor limit cycles.

We note that the Lev Ginzburg system (1) has the invariant straight line $y = 1/\beta_1$. So, system (1) has at most one limit cycle, and if it exists then it is hyperbolic. By using Poincaré Compactification, we show that the only chance to equation (1) exhibit a limit cycle is that case where the limit cycle is a non-hyperbolic one.

References

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