Study of the equilibrium points of the restricted three-body problem with an oblate primary

Márcia P. Dantas[1], Hildeberto E. Cabral[2]

[1] *Department of Mathematics, Universidade Federal Rural de Pernambuco, Dois Irmãos, 52171–900, Recife, PE, Brazil.*

[2] *Department of Mathematics, Universidade Federal de Pernambuco, Cidade Universitária, 50670–901, Recife, PE, Brazil.*
E-mail: hild@dmat.ufpe.br

We consider the restricted three–body problem with the more massive primary as an oblate spheroid. The primaries are moving in a keplerian circular orbit about their center of mass, the equatorial plane of the oblate primary coinciding with the plane of motion of the binaries.

We study the dynamics of a third particle of infinitesimal mass in space under the gravitational attraction of the binary, looking at the existence and stability of the equilibrium points.

The planar problem has been studied by several authors. They have confirmed the existence of five equilibrium points, three of them being collinear and two in a triangular configuration, as in the problem without oblateness. The collinear ones are unstable in the Liapunov sense; in the interval of linear stability of the triangular ones it has been shown stability except for three values of the mass parameter and the critical mass [5], [7], [8]. These results follow from Arnold Theorem [2].

In the spatial case, a three-degree of freedom system, Arnold Theorem does not apply but we can still try to establish stability results in a weaker formulation such as formal stability and stability of finite type. To this end we use normal forms techniques and the theory developed in [4]. We expand the potential in power series up to fourth order in the oblateness parameter, the eccentricity of the spheroid.

This spatial problem has been studied with an approximation of the potential to second order in the oblateness parameter [5], [6]. Recently, Markellos and Douskos, found two new equilibrium points outside the equatorial plane, nearly above and below the oblate primary [3]. We hope to say something about this point.

References

[1] K. B. Bhatnagar and L. M. Saha, *General motion of n–rigid bodies*, Bull. Astr. Soc. India **22** (1994), 1–19.

[2] Hildeberto E. Cabral and Kenneth R. Meyer, *Stability of equillibria and fixed points of conservative systems*, Nonlinearity **12** (1999), 1351–1362.

[3] C. N. Douskos and V. V. Markellos, *Out-of–plane equilibrium points in the restricted three–body problem with oblateness*, A&A **446** (2006), 357–360.

[4] A. P. Markev, *Libration Points in Celestial Mechanics and Astrodynamics*, MIR Publ., Moscow, 1978.

[5] Ram Krishan Sharma and P. V. Subba Rao, *Stationary solutions and their characteristic exponentes in the restricted three–body problem when the more massive primary is an oblate spheroid*, Celestial Mechanics **13** (1976), 137–149.

[6] Ram Krishan Sharma and P. V. Subba Rao, *A case of commensurability induced by oblateness*, Celestial Mechanics **18** (1978), 185–194.

[7] Ram Krishan Sharma and P. V. Subba Rao, *Effect of oblateness on triangular solutions at critical mass*, Astrophysics and Space **60** (1979), 247–250.

[8] Claudio Vidal, *Stability of equilibrium positions of hamiltonian systems*, Qual. Th. Dyn. System **7** (2008), 253–294.