Global dynamics of the May–Leonard system

Gamaliel Blé González[1], Víctor Castellanos[2], Jaume Llibre[3], Ingrid Quilantán Ortega[4]

[1] *División Académica de Ciencias Básicas, Universidad Juárez Autónoma de Tabasco, Cunduacán, Tabasco, Mexico.*
E-mail: gble@ujat.mx

[2] *División Académica de Ciencias Básicas, Universidad Juárez Autónoma de Tabasco, Cunduacán, Tabasco, Mexico.*
E-mail: vicas@ujat.mx *URL:* http://www.ujat.mx

[3] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Catalonia, Spain.*
E-mail: jllibre@mat.uab.cat

[4] *División Académica de Ciencias Básicas, Universidad Juárez Autónoma de Tabasco, Cunduacán, Tabasco, Mexico.*
E-mail: xangari82@yahoo.com.mx

We talk about the integrability and the global dynamics of the May–Leonard model in R^3, which describe the competition between three species and depending on two positive parameters a and b. Specially we analyze the cases $a+b=2$ and $a=b$ in the compactification of the positive octant. Roughly speaking, if $a+b=2$ and $a \neq 1$ there are invariant topological half–cones by the flow of the system. These half–cones have vertex at the origin of coordinates and surround the bisectrix $x=y=z$, and foliate the positive octant. The orbits of each half–cone are attracted to a unique periodic orbit of the half–cone, which lives on the plane $x+y+z=1$.

When $b=a \neq 1$ then we consider two cases. First, if $0<a<1$ then the unique positive equilibrium point attracts all the orbits of the interior of the positive octant. Second, if $a>1$ then there are three equilibrium points in the boundary of positive octant, which attract almost all the orbits of the interior of the octant, we describe completely their bassins of attractions.

References

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