On the maximum number of limit cycles of a class of generalized Liénard differential systems

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Applying the averaging theory of first, second and third order to one class generalized polynomial Liénard differential equation, we improve the known lower bounds for the maximun number of limit cycles that this class can exhibit.

More precisely, given the equation

$$\begin{aligned} \dot{x} &= y + \sum_{k\geq 1} \varepsilon^k h_l^k(x), \\ \dot{y} &= -x - \sum_{k\geq 1} \varepsilon^k (f_n^k(x)y + g_m^k(x)), \end{aligned} \tag{1}$$

where for every k the polynomials $h_l^k(x)$, $g_m^k(x)$ and $f_n^k(x)$ have degree l, m and n respectively, and ε is a small parameter. We show that the following result.

Theorem. If for every $k = 1, 2$ the polynomials $h_l^k(x)$, $g_m^k(x)$ and $f_n^k(x)$ have degree l, m and n respectively, with $l, m, n \geq 1$, then for $|\varepsilon|$ sufficiently small, the maximum number of medium limit cycles of the polynomial Liénard differential systems (1) bifurcating from the periodic orbits of the linear center $\dot{x} = y$, $\dot{y} = -x$, using the averaging theory

(a) of first order is $\tilde{H}_1(l,m,n) = \left[\dfrac{\max\{O(l), O(n+1)\} - 1}{2}\right] = \max\left\{\left[\dfrac{l-1}{2}\right], \left[\dfrac{n}{2}\right]\right\}$,

(b) of second order is $\tilde{H}_2(l,m,n) = \left[\dfrac{\max\{E(l)+E(m), O(n)+E(m)+1, O(l), O(n+1)\} - 1}{2}\right]$,

(c) of third order is $\tilde{H}_3(l,m,n) \geq \left[\dfrac{\max\{O(m+n), E(l+m)-1\} - 1}{2}\right]$, and

(d) the three upper bounds for $\tilde{H}(l,m,n)$ given in statements (a), (b) and (c) for some values of l, m and n are reached. So they cannot be improved.

Where $E(k)$ is the largest even integer $\leq k$, and $O(k)$ is the largest odd integer $\leq k$.