Behavior of the binary collision in a planar restricted (N+1)-body problem

Martha Alvarez-Ramírez[1], Claudio Vidal[2]

[1] *Departamento de Matemáticas UAM–Iztapalapa, 09340 Iztapalapa, México, D.F., Mexico.*
E-mail: mar@xanum.uam.mx

[2] *Departamento de Matemática, Facultad de Ciencias, Universidad del Bío–Bío, Casilla 5-C, Concepción, VIII-Región, Chile.*
E-mail: clvidal@ubiobio.cl

We consider the planar restricted $(N+1)$-body problem, where the primaries are moving in a central configuration. It is verified that when the energy approaches minus infinity, then the infinitesimal mass m_1 is close enough to a primary, we use Levi-Civita and McGehee coordinates to regularize the binary collision. A canonical transformation is constructed which leaves the equations in the form of a perturbed resonant pair of harmonic oscillators where the perturbation parameter is the reciprocal of the energy. Firstly, it is proved the existence of four transversal ejection-collision orbits. After that the construction of the annulus mapping is carried out, and the condition of the Moser invariant curve theorem are verified, and then we are able to infer the existence of long periodic solutions for the restricted $(N+1)$-body problem. Also it is proved the existence of quasi-periodic solutions close to the binary collision. The first result implies, via KAM theorem, the existence, for certain intervals of values of the Jacobi constant, of an uncountable number of invariant punctured tori in the corresponding energy surface.

This work grew out of an attempt to carry over the methods of the study of the restricted three body problem for high values of the Jacobian constant by Conley [1], Chenciner [2] and Chenciner-Llibre [3] applying their techniques to a more general restricted problem. Our goal in this paper is to give a generalization of the Conley thesis results. In addition , we show that the Hill terms (the terms of sixth order) are of the same nature but with different coefficients, which allow us to give the differences with respect to known results. Thus we point out conditions on the relative equilibrium of the N-body problem in order to overcame the difficulties.

References

[1] Conley, C. *On some new long periodic solutins of the plane restricted three body problem*, *Comm. Pure Appl. Math.* **16** (1963), 449–467.

[2] Chenciner, A., *Le probl̀eme de la lune et la théorie des syst̀emes dynamiques*, *Preprint.* Universitè Paris VII.

[3] Chenciner, A.; Llibre, J., *A note on the existence of invariant punctured tori in the planar circular restricted three-body problem*, Ergodic Theory Dynam. Systems **8*** (1988), Charles Conley Memorial Issue, 63–72.