SEMICLASSICAL QUANTIFICATION OF SOME TWO DEGREE OF FREEDOM POTENTIALS: A DIFFERENTIAL GALOIS APPROACH

Primitivo Acosta-Humánez
Instituto & Escuela de Matemática
Universidad Autónoma de Santo Domingo
Dominican Republic
pacosta-humanez@uasd.edu.do

J. Tomás Lázaro
Departament de Matemàtiques
Universitat Politècnica de Catalunya (UPC)
Centre de Recerca Matemàtica (CRM)
Institute of Mathematics of the UPC-BarcelonaTech (IMTech)
Barcelona, Spain
jose.tomas.lazaro@upc.edu

Juan J. Morales-Ruiz
Departamento de Matemática Aplicada
Universidad Politécnica de Madrid (UPM)
Madrid, Spain
juan.morales-ruiz@upc.edu

Chara Pantazi
Departament de Matemàtiques
Universitat Politècnica de Catalunya (UPC)
Barcelona, Spain
chara.pantazi@upc.edu

ABSTRACT

In this work we explain the relevance of the Differential Galois Theory in the semiclassical (or WKB) quantification of some two degree of freedom potentials. The key point is that the semiclassical path integral quantification around a particular solution depends on the variational equation around that solution: a very well-known object in dynamical systems and variational calculus. Then, as the variational equation is a linear ordinary differential system, it is possible to apply the Differential Galois Theory to study its solvability in closed form. We obtain closed form solutions for the semiclassical quantum fluctuations around constant velocity solutions for some systems like the classical Hermite/Verhulst, Bessel, Legendre, and Lamé potentials. We remark that some of the systems studied are not integrable, in the Liouville - Arnold sense.

Keywords Quantification, path integrals, propagator, semiclassical approximation, differential Galois theory, integrability.

Introduction

In [18] the third author suggested the relevance that the Differential Galois Theory could play in the Feynman's path integral approach in Quantum Mechanics. Indeed, the key proposal was to study whether it was possible to obtain, in closed form, the semiclassical approximation of the Feynman's propagator K, see [8, 22].

Let us recall the notation and main ideas in [18]. Given n the number of degrees of freedom, we denote by $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ the position, t the time, and γ is a path from $(\mathbf{x}_0, t_0)$ to $(\mathbf{x}_1, t_1)$. This classical path γ in the configuration space defines an integral curve Γ in the phase space, assuming there are non focal (conjugated) points (chapter 9 of [6]).

The computation of the propagator $K(\mathbf{x}_1, t_1 \,|\, \mathbf{x}_0, t_0)$ around the path γ in the semiclassical approach (where $\hbar$ is small) can be obtained through

$$K(\mathbf{x}_1, t_1 \,|\, \mathbf{x}_0, t_0) = K_{\mathsf{WKB}} \left(1 + \mathrm{O}(\hbar)\right),$$