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A WEAKENED MARKUS–YAMABE CONDITION FOR PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS OF DEGREE $(1, n)$

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Abstract For a general autonomous planar polynomial differential system, it is difficult to find conditions that are easy to verify and which guarantee global asymptotic stability, weakening the Markus–Yamabe condition. In this paper, we provide three conditions that guarantee the global asymptotic stability for polynomial differential systems of the form $x' = f_1(x, y)$, $y' = f_2(x, y)$, where f_1 has degree one, f_2 has degree $n \geq 1$ and has degree one in the variable y. As a consequence, we provide sufficient conditions, weaker than the Markus–Yamabe conditions that guarantee the global asymptotic stability for any generalized Liénard polynomial differential system of the form $x' = y$, $y' = g_1(x) + yg_2(x)$ with g_1 and g_2 polynomials of degrees n and m, respectively.

Keywords: global asymptotic stability; Markus–Yamabe conjecture; planar polynomial vector fields

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1. Introduction and statement of the main results

Since the time of Liapunov, it has become evident that finding conditions that guarantee global asymptotic stability of an equilibrium point in a differential system, even in two dimensions, is a difficult problem. Liapunov's approach is probably the most widespread general method used, though constructing a Liapunov function usually requires ingenuity, experience and some luck. For the two-dimensional autonomous system

$$x' = f_1(x, y), \qquad y' = f_2(x, y), \tag{1}$$

with $f = (f_1, f_2)\colon \mathbb{R}^2 \to \mathbb{R}^2$, we seek for a set of easily verifiable conditions on the function f which can give global asymptotic stability. A result to this end was proven in