# ANALYTIC STUDY OF TWO LIMIT CYCLES BIFURCATING FROM A ZERO–HOPF EQUILIBRIUM

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Abstract. In this paper we provide sufficient conditions for the existence of two limit cycles bifurcating from the unique zero–Hopf equilibrium of the differential system

$$\dot{x} = y + a, \quad \dot{y} = -x + z, \quad \dot{z} = -bx^2 + z^2 + c,$$

where $a$, $b$ and $c$ are real arbitrary parameters. Our study uses the averaging theory.

## 1. Introduction and Statement of the Main Results

Differential systems appear in many areas, such as, Physics, Biology, Chemistry, Control Theory, Mechanical Oscillators, ... see for instance [1, 2, 12], and their study is important. A limit cycle of a differential system is a periodic orbit isolated in the set of all periodic orbits of the differential system. The study of the limit cycles of polynomial differential systems has two main motivations, first they play a relevant role in the dynamics of the differential systems, and also their analysis become motivated by Hilbert's 16-th problem, see [4, 11].

A way to produce limit cycles is perturbing a differential system having zero–Hopf equilibria with a real small parameter $\varepsilon$. An equilibrium point of a differential system in $\mathbb{R}^3$ such that the eigenvalues of the Jacobian matrix of the differential system at the equilibrium point are $0$ , $\omega\, i$ and $-\omega\, i$, with $\omega > 0$ is called *zero–Hopf equilibrium*. For more details regarding on the zero–Hopf bifurcations see [5, 6, 10].

In the paper [7] the authors investigated the chaotic behavior of the differential system

$$\begin{aligned} \dot{x} &= y + a, \\ \dot{y} &= z - x, \\ \dot{z} &= z^2 - bx^2 + c, \end{aligned} \tag{1}$$

when it has no equilibria, i.e. when either $(b-1)c < 0$, or $b = 1$. Differential system (1) also is the differential system (21) of the page 64 of the book [8].

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