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ORIGINAL PAPER

Limit Cycles for Discontinuous Piecewise Differential Systems in $\mathbb{R}^3$ Separated by a Paraboloid

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Abstract
In planar piecewise differential systems it is known that when the discontinuity curve is a straight line and both differential systems are linear centers, these piecewise differential systems have no limit cycles but if they are separated by other types of discontinuity curves, such as parabolas, then they have limit cycles. All these results are in the plane and although the qualitative theory of planar piecewise differential systems has been the subject of many research, this is not the case for piecewise differential systems in higher dimensions. In this paper, we study the maximum number of limit cycles of discontinuous piecewise differential systems in $\mathbb{R}^3$ separated by a paraboloid (elliptic or hyperbolic), and formed by what we call two linear differential centers. We prove that these systems can have at most one limit cycle and that this upper bound is reached. We also provide systems of these types without periodic solutions and with a continuum of periodic solutions.

Keywords Discontinuous piecewise linear systems · Limit cycles · First integrals

Introduction and Statement of the Main Result

The study of piecewise linear differential systems goes back to Andronov et al. [1] and still continues to receive attention from researchers. These last years a renewed interest has appeared in the mathematical community for understanding the dynamical richness of

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