LIMIT CYCLES IN A CLASS OF PLANAR DISCONTINUOUS PIECEWISE QUADRATIC DIFFERENTIAL SYSTEMS WITH A NON-REGULAR DISCONTINUOUS BOUNDARY (II)

DONGPING HE[1, ✉] AND JAUME LLIBRE[2]

Abstract. In our previous work we have already studied the Poincaré bifurcation for a class of discontinuous piecewise quadratic polynomial differential systems with a non-regular discontinuous boundary, which is formed by two rays starting from the origin and forming an angle $\alpha = \pi/2$. The unperturbed system is the quadratic uniform isochronous center $\dot{x} = -y + xy$, $\dot{y} = x + y^2$ with a family of periodic orbits surrounding the origin. In this paper, we continue to investigate this kind of piecewise differential systems but now the angle between the two rays is $\alpha \in (0, \pi/2) \cup [3\pi/2, 2\pi)$. Using the averaging theory of first order and the Chebyshev theory we prove that the maximum number of hyperbolic limit cycles which can bifurcate from these periodic orbits is exactly 8 for $\alpha \in (0, \pi/2) \cup [3\pi/2, 2\pi)$. Together with our previous work, which concerns on the case of $\alpha = \pi/2$, we can conclude that by using the averaging theory of first order the maximum number of hyperbolic limit cycles is exactly 8, when this quadratic center is perturbed inside the above mentioned classes separated by a non-regular discontinuous boundary with $\alpha \in (0, \pi/2] \cup [3\pi/2, 2\pi)$.

1. Introduction and statement of the main results

The *weak Hilbert's* 16*th problem* proposed by Arnold [2], that concerns about the maximum number of orbits of the centers of polynomial differential systems that persist as limit cycles (i.e., isolated periodic orbits) when they are perturbed inside the class of all polynomial differential systems of degree n. This is one of the classical ways to produce limit cycles, and this mechanism is also called Poincaré bifurcation. Although the weak Hilbert's 16th problem reduces the difficulty of the investigation of the second part of the *Hilbert's* 16*th problem*, which ask for an upper bound of the maximum number of limit cycles in function of the degree of the planar polynomial differential systems, and for the possible distribution of the limit cycles, see [16, 19, 21]. The possible distributions of limit cycles has been solved, see [31], but to find such upper bound remains unsolved.

In this paper we will perturb a uniform isochronous center. Recall that for planar polynomial differential systems, Conti [10] proved that a center is called a *uniform isochronous center* if in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$ it can be written in the form $\dot{r} = R(\theta, r), \dot{\theta} = k$, where k is a nonzero real number.

As it is indicated in [27] the quadratic polynomial differential systems with a uniform isochronous center can be written as,

$$\dot{x} = -y + xy, \quad \dot{y} = x + y^2. \tag{1}$$

In fact, this system, under the transformation $x = -Y, y = X$, can be transformed into the system (S_2) of Table 4 in [7], or of Table 1 in [33]. Therefore systems (1) and (S_2) are equivalent, and then we will summarize the existing results for system (1) even if some of them are concerned on system (S_2).

2020 *Mathematics Subject Classification.* Primary: 34C05, 34C07, 34C25, 34C29.

Key words and phrases. Limit cycle, discontinuous piecewise polynomial system, quadratic uniform isochronous center, non-regular discontinuous boundary, averaging theory, Chebyshev theory.

✉ Corresponding author.