FROM ABEL'S DIFFERENTIAL EQUATIONS TO HILBERT'S 16TH PROBLEM

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Dedicado a mi profesor y amigo Jorge Sotomayor, quien nos dejó demasiado pronto
Dedicated to my professor and friend Jorge Sotomayor, who left us too soon

Abstract. The study of the limit cycles of planar polynomial differential equations is motivated both by its appearance in many mathematical models of the real-world as for the second part of Hilbert's 16th problem. In this work we briefly summarize some results on this subject and we will also highlight the important role that the Abel's differential equations play in its study. In the way, we recall some nice properties of the Riccati's differential equations.

1. Introduction

The aim of this work is to motivate the study of limit cycles of planar autonomous ordinary differential equations (DEs) and to illustrate the difficulties of their study. To this end, we will present some simple real-world problems where limit cycles appear and we will also recall the Hilbert's 16th problem.

We will focus our attention on the limit cycles for Abel's DEs because this family is perhaps the "easiest one" where this question is open. Moreover, it is known that the Hilbert's problem restricted to the DEs of degree 2 will follow from the full knowledge of Abel's equations. In the way we will collect related results for linear and Riccati's DEs.

I first heard of Abel's equations while I was doing my Ph.D. thesis under the supervision of Jaume Llibre. During that period we collaborated with Jorge Sotomayor and, in Chapter 4 of this thesis, we used them to control the number of limit cycles of a family of planar DEs, see [58, 69, 70].

This work is an updated and extended version of my paper [60], which was published in Catalan, and it was based on the opening lecture of the academic year 2011-12 at the Department of Mathematics of the Universitat Autònoma de Barcelona. The title of that lesson was "Equacions diferencials d'Abel o el miratge de la simplicitat", that is *Abel's differential equations or the mirage of simplicity.*

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